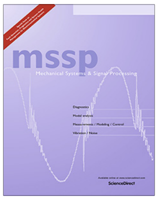
Mechanical Systems and Signal Processing 115 (2019) 696–719

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp



On the contact interaction of a two-layer beam structure with clearance described by kinematic models of the first, second and third order approximation

V.A. Krysko [a], J. Awrejcewicz [b,*], O.A. Saltykova [a,c], I.V. Papkova [a], A.V. Krysko [d,c]

[a] Department of Mathematics and Modelling, Saratov State Technical University, Politehnicheskaya 77, 410054 Saratov, Russian Federation
[b] Department of Automation, Biomechanics and Mechatronics, Lodz University of Technology, 1/15 Stefanowskiego Str., 90-924 Lodz, Poland
[c] Cybernetic Institute, National Research Tomsk Polytechnic University, 30 Lenin Avenue, 634050 Tomsk, Russian Federation
[d] Department of Applied Mathematics and Systems Analysis, Saratov State Technical University, Politehnicheskaya 77, 410054 Saratov, Russian Federation

ARTICLE INFO

Article history:
Received 24 January 2018
Received in revised form 13 June 2018
Accepted 23 June 2018

Keywords:
Beams
Vibrations
Clearance
Non-linearity
Chaos
Finite difference method

ABSTRACT

In this article, the contact interaction of two geometrically non-linear beams with a small clearance is studied by employing various approximations of the kinematic beam models. Our investigation concerns two-case studies, i.e. when the upper beam is governed by the second order Timoshenko model (i) or by third order Pelekh-Sheremetev-Reddy-Levinson model (ii), whereas in both cases the lower beam is described by the kinematic (Bernoulli-Euler) model of the first approximation.

The upper beam is subjected to the transversal, uniformly distributed harmonic load, whereas the beam interaction follows the classical Kantor's model. The problem is highly non-linear due to occurrence of the geometric von Kármán non-linearity and the contact interaction between beams (structural non-linearity). The governing PDEs are reduced to ODEs by the method of finite differences (FDM) of the second order. The obtained ODEs are solved by a few Runge-Kutta type methods of different orders. Results of convergence versus the number of partition points along the spatial co-ordinate and time steps are investigated.

New non-linear phenomena of the studied structural package are detected, illustrated and discussed with emphasis on the "true" chaotic vibrations. In particular, three qualitatively different algorithms for computation of largest Lyapunov exponents are employed, and the differences between geometrically linear versus non-linear problems are reported.

1. Introduction

The classical beams, and more recently the layered/sandwich beams, are widely used as construction elements in numerous branches of industry like the air-planes and space industries, civil engineering, mechanical engineering, biomechanical engineering, MEMS fabrication, and so on.

There are numerous examples of the theoretical and engineering oriented analysis of the uniform/non-uniform layered beams within the statics/dynamics as well as linear/non-linear studies. Structural engineering exhibits vast class of possible

---

* Corresponding author.
E-mail addresses: tak@san.ru (V.A. Krysko), awrejcew@p.lodz.pl (J. Awrejcewicz), olga_a_saltykova@mail.ru (O.A. Saltykova), ikravzova@mail.ru (I.V. Papkova).

https://doi.org/10.1016/j.ymssp.2018.06.045
0888-3270/

fabrications of multi-layer-beams including beams with fiber reinforced polymers, steel-concrete and wood-concrete beams etc.

In the case of the MEMS fabrication, the traditional micro-systems are supplemented by micro-beams. During the process of exploitation the beams are subjected to numerous and various excitations including different mechanical loads (static, dynamic, uniform and non-uniform, short or long term time-dependent actions coming from various sources, etc.), temperature fields, electromagnetic fields as well as noisy environments.

The fast development of industrial processes in the field of nano-, micro- and macro-sciences requires a proper feedback from the classical mechanics and applied mathematics to guarantee reliable and high accuracy mathematical models of the structural members which can be successfully employed in different real world scales and adjusted to challenging industrial needs.

Modelling and identification of delamination in double-layer Bernoulli-Euler beams has been studied by Orłowska et al. [1]. A concept of the contact layer, assuming simple truss connections without friction, has been supported by the experimental validation of the theoretical background. Two co-existing delamination zones have been detected.

A finite element method has been employed to study the coupling of an Bernoulli-Euler beam with a Timoshenko steel beam for the reinforced concrete slab, and then by taking the Timoshenko kinematic hypothesis for both concrete slab and steel beams by Ranzi and Zona [2,3].

The Timoshenko kinematic hypothesis for each beam component, while taking into consideration the constraint of equal cross-section rotation for both layers, has been used by Xu and Wu [4] to construct an analytical model for static, dynamic and buckling analysis.

Nguyen et al. [5] reported a full closed-form solution for shear-deformable two-layer beams with distributed bond including the explicitly derived exact stiffness matrix.

The linear static analysis of shear-deformable two-layer beams with interlayer slip, taking into account the Timoshenko beam and the layer located in a discontinuous way, has been carried out by Nguyen et al. [6]. A simple dry friction model has been introduced and the exact stiffness matrix for a generic two-layer beam elements has been derived. A parametric study dealing with a two-span layered composite beam subjected to uniformly distributed load has been carried out.

Two-layer beams with interlayer slip and bi-linear interface line have been studied by Campi and Monetto, by using the Timoshenko hypotheses [7]. The layers we assumed to be perfect/imperfect in the transverse/longitudinal direction, and the interlayer slips were allowed. The problem has been solved analytically and explicit formulas for all static and kinematic variables have been reported.

Dynamic behavior including a study of chaotic vibrations of two-layer beams with frictional interface and stick-slip phenomena has been analyzed by Sedighi et al. [8]. Coupled differential equations governing transverse and longitudinal vibration of two-layers, in the presence of dry frictions, have been derived and a parametric numerical study supported by the Poincaré maps and the Lyapunov exponents have been carried out.

While taking into account the tangential and transverse strains in the interlayer, the elasticity solution of a simply supported two-layer beam subjected to time-dependent load has been proposed by Wu et al. [9]. It has been shown that the obtained results are in accordance with the 2D finite element solutions but they are different from the Bernoulli-Euler solution for thick beams. The effect of interlayer thickness and viscoelastic properties on displacements and stresses of the beam have been illustrated and discussed.

This paper is based on two main features. First, the object of investigation is *real world/engineering oriented* since it deals with vibrations of two beams, separated by the clearance and externally harmonically loaded. This physical model can be found in various engineering constructions/devices of the aforementioned industrial branches.

The second feature of our study refers to purely scientific results associated with a study of non-linear mechanical systems governed by non-linear PDEs and the Kantor model of contact problems, i.e. it is within the framework of the qualitative theory of differential equations, dynamical systems and mechanics.

We are aimed at a study of the problem as an infinite one being governed by PDEs, and this is why we are focused rigorously on the reliability of the results, while reducing the problem to finite dimension, i.e. to the Cauchy problem regarding a system of infinite number of degrees-of-freedom. In other words, the introduced truncation of the obtained set of ODEs should validate our results as reliable and applicable to the case of infinite dimension, i.e. to the governing PDEs of the original systems.

In order to satisfy the reliability of the solution of the formulated non-linear PDEs, we need to employ qualitatively different numerical methods and validate the obtained solutions by comparison of the simulation results yielded by those usually qualitatively different methods.

It is well known that the sensitivity of the initial conditions is one of the fundamental features of chaos. According to a definition of chaos given by Devaney [10], chaos has the following properties: (i) dependence on the initial conditions; (ii) the intermittency/transitivity conditions; (iii) regular density of periodic orbits. Banks et al. [11] have proved that the property of being dependent on the initial conditions can be removed. According to Knudsen's definition of chaos, a function given on a bounded metric space can be defined as chaotic one if it has got a dense orbit and exhibits the essential dependence on the initial conditions [12].

Gulick [13] claims that chaos exists if there is either a strong dependence on the initial conditions or a time signal has a positive Lyapunov exponent in each point of the studied space, and hence it does not exhibit any regular behaviour. In this work we follow the definition of chaos introduced by Gulick [13].