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Predicting remaining useful life based on a generalized degradation with fractional Brownian motion

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A B S T R A C T

For data-driven remaining useful life (RUL) prediction, an appropriate degradation model is critically important to achieve accurate prediction. The degradation processes in some practical systems are not only related to the age but also related to the current degradation state, and the degradation processes may be non-Markovian processes. However, most existing stochastic process-based degradation models only depend on the age, and simply assume that the increments are independent. In this paper, an age- and state-dependent degradation model with long-range dependence is developed, which is more general than most of the existing models based on either Brownian motions (BMs) or fractional Brownian motions (FBMs). The Radon-Nikodym derivative is utilized to obtain a likelihood ratio function of unknown parameters, and the estimates are obtained by maximizing the likelihood ratio function. A weak convergence theorem is introduced to approximate the FBM by a BM with a time-varying coefficient. A time-space transformation is further utilized to obtain an approximate explicit solution of the RUL. At last, numerical simulations and two real case studies of blast furnace walls and ball bearings are adopted to verify the effectiveness of the proposed model.

1. Introduction

As one of the key issues of prognostics and health management (PHM), the prediction of remaining useful life (RUL) can provide useful information for the maintenance activities arranging and spare part provisioning [1–3]. To achieve an accurate prediction on the RUL distribution, an appropriate degradation model that fully characterizes the actual degradation process is required. As a consequence, many research efforts have been made centred on the degradation modeling in the past decades [4]. In general, the degradation process is often modeled as the sum of a non-random drift and a random diffusion. The former describes the inherent characteristics of the degradation, while the later describes the randomness, caused by environments, operations, interference and so on. Up to now, Wiener process-based models are extensively adopted to describe the degradation processes, because they possess favourable mathematical characteristics and reasonable physical interpretations [4–7]. For a class of degradation processes with linear characteristic, Doksum et al. [8] developed a linear degradation model based on Brownian motion (BM). To expand the scope of application of this model, Whitmore et al. [9] and Gebraeel et al. [10] proposed preprocessing methods for some nonlinear degradation data based on time-scale

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transformation and logarithmic transformation, respectively. Further, Si et al. [11] developed a general nonlinear degradation model with unit-to-unit variability. Based on this model, Wang et al. [12] introduced a time-scale transformation in BM. This kind of degradation models usually includes an age-dependent drift item and a standard BM with a diffusion coefficient, and is also a Markov process with independent increments.

However, in practical systems, the degradation states may influence the rate of degradation, because the resistance to deteriorate might be changed with the degradation [13]. According to some relevant literature, such characteristic has been discovered in the degradation processes of ball bearings [13], cylinder liners [14], and metal alloys [15]. Along this direction, few research efforts were carried out to deal with the RUL prediction of such degradation processes [13,14,16–18]. Giorgio et al. [16] proposed a degradation model based on Markov chain, in which the transition probabilities between process states depend on the current system state. Based on this model, Giorgio et al. [15] further proposed a new four-parameter Markov chain to characterize the state-dependence. Guida et al. [17] proposed a continuous-state discrete-time Markov model, in which the Pearson's family of distributions are used to approximate the true transition density. Based on Wiener process, Zhang et al. [13] and Li et al. [18] studied an age- and state-dependent degradation process. However, the degradation models in these works base either on Markov chains or BM, which are all Markov processes.

Due to the influence of the environments, operating conditions, and even the degradation mechanism itself, some practical systems cannot meet the Markov property. Real data revealing long-range dependence has been reported, such as variable-bit-rate video traffic [19], network data streams [20], trade data on the stock market [21], and DNA sequences [22]. Moreover, long-range dependence has also been reported in the degradation processes of turbofan engines [23], blast furnace walls [24], and Li-Ion batteries [25]. However, the RUL prediction of degradation processes with long-range dependence has been scarcely considered in the literature. Long-range dependence implies that the dependence among nonoverlapping increments of the process decays slowly with increasing distance [26]. It is noted that the long-range dependence is usually measured by a Hurst exponent [27]. In [23], a degradation model with long-range dependence is developed by an FBM-based process, and an RUL prediction based on numerical simulation is obtained. Then, in our previous work, an approximate explicit PDF of the FPT of such degradation is achieved [24]. However, these works just considered the age-dependent processes, which assume that the degradation rate of the equipment depends entirely on its current age. This assumption limits the scope of application of the methods given in [23,24]. In fact, there are many equipment that do not satisfy this assumption, such as bearings and blast furnace walls. For example, considering the wear process of the bearing, the degree of wear changes its vibration frequency, amplitude and other characteristics, and the vibration of the bearing in turn affects its wear rate. For such equipment, the methods in [23,24] hardly give satisfactory results.

To sum up, in the existing literature, efforts have been made to predict the RUL for either age- and state-dependent degradation, or long-range dependent degradation. However, to the best of our knowledge, the existing works still lack an effective prediction method that can simultaneously consider the challenges from age- and state-dependent, and long-range dependent degradation. In this paper, a generalized degradation model is developed, in which age- and state-dependence, as well as long-range dependence are included. In this model, the age- and state-dependence is described by an age- and state-dependent drift coefficient, while the long-range dependence is embodied by an FBM [28]. In fact, FBM is an extension of BM in the fraction dimension. When the Hurst exponent H satisfying $\frac{1}{2} < H < 1$, an FBM is a long-range dependence process [29]. To estimate the unknown parameters in the degradation model, we obtain the likelihood ratio function by the Radon-Nikodym derivative of the measurement transformation. Due to the long-range dependence, an FBM is neither a semimartingale nor a Markov process when the Hurst exponent $H \notin \{\frac{1}{2}, 1\}$. So the analytical form of the RUL is intractable to deduce directly by solving the Fokker-Planck-Kolmogorov (FPK) equation. To obtain an approximate explicit solution of the PDF of RUL, we apply a weak convergence theorem and a time–space transformation to transform the first passage time (FPT) of the age- and state-dependent degradation with long-range dependence into the FPT of a BM. In order to verify the effectiveness of the proposed method, three kinds of degradation models and practical cases of blast furnace walls and ball bearings are presented.

The main contributions of this paper are summarized as: (i) A novel degradation model is developed, which simultaneously considers age- and state-dependence, as well as long-range dependence. The developed model is more general than most of the existing degradation models. (ii) To deal with the parameter estimation, the Radon-Nikodym derivative is adopted to obtain a likelihood ratio function. The measurement transformation is adopted to transform the original degradation process into a semimartingale with independent increments. Compared with the estimation methods used in [23,24], this method can avoid high-dimensional matrix inversion operations, thereby reducing the amount of computation. (iii) The FPT of age- and state-dependent processes with long-range dependence is derived in an explicit form using a time–space transformation. This result includes the RUL of the age-dependent degradation processes with long-range dependence studied in [23,24] as a particular case. The practical cases of blast furnace walls and ball bearings show that, the proposed model achieves more satisfactory results than the degradation models neglecting the age- and state-dependence or the long-range dependence.

The rest of this paper is structured as follows: In Section 2, a generalized degradation model is developed to describe the age- and state-dependent degradation with long-range dependence. A parameter estimation method based on the Radon-Nikodym derivative is introduced in Section 3. An approximate explicit PDF of RUL of the age- and state-dependent degradation with long-range dependence is deduced in Section 4. In Section 5, three numerical examples illustrate the validity of