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Brief paper

Spatio-temporal multi-robot routing[☆]

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ABSTRACT

In this paper, we consider the problem of routing multiple robots to service spatially distributed requests at specified time instants. We show that such a routing problem can be formulated as a pure assignment problem. Additionally, we incorporate connectivity constraints into the problem by requiring that range-constrained robots ensure a connected information exchange network at all times. We discuss the feasibility aspects of such a *spatio-temporal* routing problem, and derive the minimum number of robots required to service the requests. Moreover, we explicitly construct the corresponding routes for the robots, with the total length traveled as the cost to be minimized.

1. Introduction

Multi-robot routing requires multiple robots to visit a set of spatially distributed locations for some purpose (e.g., delivery or acquisition) with routes that optimize certain criteria (e.g., minimization of total distance traveled, completion time, or energy consumption). In this paper, we consider such a problem of servicing spatial requests, with an added temporal constraint that each request be serviced at a specified time instant. Moreover, we consider the connectivity constrained version of the problem, where we require that the underlying information exchange network remains connected at all times.

In the robotics literature, multi-robot routing is a well studied topic (e.g. see Burgard, Moors, Stachniss, & Schneider, 2005, and Mosteo, Montano, & Lagoudakis, 2008). Many problems on combinatorial optimization are associated with multi-robot routing. For instance, the multiple traveling salesman problem (m-TSP) consists of determining a set of optimal routes for m salesmen who all start from and turn back to a home city (see Bektas, 2006). Another example is the vehicle routing problem (VRP) (see Arsie, Savla, & Frazzoli, 2009), which concerns the design of optimal delivery or collection routes for a fleet of vehicles from one or many depots to a number of geographically scattered customers with known demands. The dynamic counterpart of the VRP, known as the dynamic vehicle routing problem, deals with online arrival of customer demands during the operation (see Bullo, Frazzoli, Pavone, Savla, & Smith, 2011, and Pavone & Frazzoli, 2010).

Applications of such routing problems include surveillance, search and rescue, transportation on demand, and assembly. However, to solve these problems is computationally expensive. In fact, the VRP is proven to be NP-hard (see Karp, 1972). To overcome this complexity, one can note that many times, applications require an ordered sequence in which requests be serviced. For instance, an autonomous structure assembly system, or a car manufacturing system, may require multiple robots to service locations in a synchronized and sequenced manner, thus motivating the need for *spatio-temporal* requests in lieu of spatial requests. In this paper, we show that by adding such temporal constraints to the spatial requests, a notion of directionality appears in the otherwise NP- hard problem of routing, and thus, it can be converted to an assignment problem, solvable in polynomial time (for preliminary results in this direction, see Chopra & Egerstedt, 2012b).

An important aspect of multi-robot coordination concerns connectivity maintenance, where in order to ensure that the robots can execute a mission in a collaborative manner, the induced information exchange network must be sufficiently rich. In this paper, we require that the range-constrained network induced by the positions of the robots be connected for all times (for preliminary results, see Chopra & Egerstedt, 2012a). In general, connectivity maintenance in multi-robot networks requires techniques

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for ensuring connectivity of a range constrained multi-robot network during some task execution. Such techniques include using relays dedicated towards maintaining sensing or communication links (e.g. Dixon & Frew, 2009, and Nguyen, Pezeshkian, Raymond, Gupta, & Spector, 2003), or using formation control strategies towards motion planning (see Kan, Dani, Shea, & Dixon, 2011). Other methods seek connectivity at *particular* time instants only, (e.g. Ponda, Johnson, Choi, & How, 2011). However, we are interested in constructing routes that maintain connectivity *for all times*, while allowing dynamic assignment between robots and *spatio-temporal* requests such that no robots exist *solely* for the task of maintaining connectivity links.

This paper is organized as follows: In Section 2, we discuss the Unconstrained Routing Problem, followed by its corresponding Connectivity Constrained version in Section 3. Finally, we demonstrate the routing problems through simulations and hardware implementations, in Section 4.

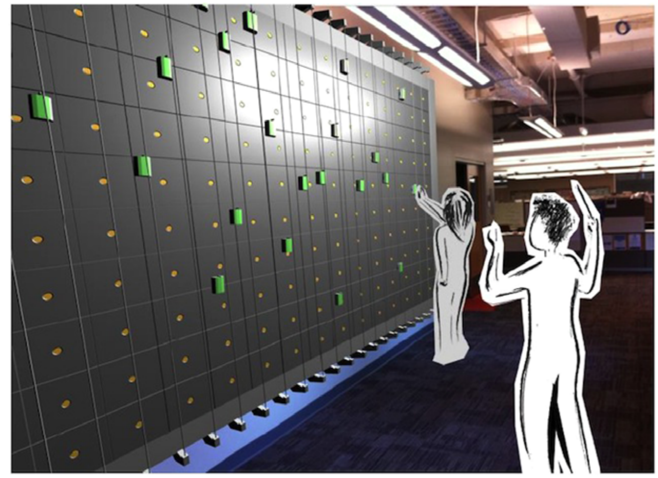

Fig. 1. A rendering of the *Robot Music Wall* concept.

A motivating example—the robot music wall

Consider a two-dimensional magnetic-based surface (wall) with a grid of strings in different pitches that generate sound when plucked. Distinct positions on the wall correspond to distinct sound frequencies, i.e. distinct notes of an instrument. Multiple robots with the ability to traverse the wall can reach these positions and pluck at the strings above them (see Fig. 1).

With this set-up, we can interpret any piece of music consisting of a series of notes to be played at specified time instants, as a series of corresponding *spatio-temporal* requests (timed positions) on the music wall. We call such a series a *Score*, which contains positions that must be reached at specified time instants. By routing multiple robots to service such timed positions, we can effectively "play" the piece of music associated with them on the wall.

2. The unconstrained routing problem

We let $T = \{t_1, t_2, \ldots, t_n\}$ denote the set of n discrete time instants over which the *Score* is defined, where $t_1 < \cdots < t_n$. Moreover, we let P_i denote the corresponding set of planar positions that require simultaneous servicing at time t_i. Each position in this set is denoted by $P_{i,\alpha}$, where $\alpha \in \{1, \ldots, |P_i|\}$ (the symbol $|\cdot|$ denotes cardinality), i.e.,

$$P_i = \{P_{i,\alpha} \mid \alpha \in \{1, \ldots, |P_i|\}\}, \quad \forall i \in \{1, \ldots, n\}. \tag{1}$$

We let $\mathcal{K}$ be the maximum number of positions that require simultaneous servicing at any time instant in T, i.e.,

$$\mathcal{K} = \max_{i \in \{1,\ldots,n\}} |P_i|. \tag{2}$$

Definition 1. Let the *Score*, denoted by *Sc*, be the set of all timed positions that the robots must reach. We express such timed positions as (position, time) pairs in the *Score*, i.e.,

$$Sc = \{(P_{i,\alpha}, t_i) \mid i \in \{1, \ldots, n\}, \alpha \in \{1, \ldots, |P_i|\}\}. \tag{3}$$

Moreover, for a given set of r robots, denoted by $R = \{1, \ldots, r\}$, we let $P_0 = \{P_{0,\alpha} \mid \alpha \in \{1, \ldots, |P_0|\}\}$ be the set of their initial positions, defined at time instant t_0.

Notice that if we have fewer robots than the maximum number of positions requiring simultaneous servicing in the *Score*, given by $\mathcal{K}$, then all $\mathcal{K}$ positions cannot be reached simultaneously. Thus, we must have at least $\mathcal{K}$ robots, i.e. $r \geq \mathcal{K}$.

We are interested in the problem of optimally routing these robots to reach the timed positions contained in the *Score*. By optimal, we mean a routing plan that minimizes the total distance traveled by the robots. Moreover, we want our solution to act at a high enough level of abstraction so that the dynamics of the robots do not have to be explicitly accounted for. This construction must be inherently hybrid in that it connects the continuous dynamics to a discrete solution. Hence, we assume single integrator dynamics for every robot, given by $\dot{x}_p = u_p, p \in R$. Since for such systems, minimum distance paths are straight lines and minimum energy motions have constant velocities, we let robots move between assigned positions in straight line paths with constant velocities that ensure their timely arrival.

Note that we can interpret the path of any robot as a *series of individual assignments* between timed positions assigned to that robot, directed in increasing order of specified time instants. Hence, the information contained in the optimal paths of the robots can be encoded in a different function that explicitly describes such individual assignments. We elaborate on this in subsequent paragraphs,

Definition 2. Let the *Assignees*, denoted by *As*, be the set containing all timed positions in the *Score* specified before the last time instant t_n, in addition to all timed initial positions of the robots, i.e.,

$$As = \{(P_{i,\alpha}, t_i) \mid i \in \{0, \ldots, n-1\}, \alpha \in \{1, \ldots, |P_i|\}\}. \tag{4}$$

Note that $r \geq \mathcal{K}$ implies that $|As| \geq |Sc|$.

We let $\pi : As \to Sc$ be a function that maps between timed positions in the *Assignees* and the *Score*. If there exists some $As' \subseteq As$, such that firstly, the restricted function $\pi|_{As'} : As' \to Sc$ is a bijection, and secondly, $\pi((P_{i,\alpha}, t_i)) = (P_{j,\beta}, t_j) \in Sc \Rightarrow t_j > t_i$ for all $(P_{i,\alpha}, t_i) \in As'$, then we call this restricted function a *feasible assignment*. The first condition ensures that every timed position in the *Score* is assigned, no two timed positions in the *Assignees* map to the same timed position in the *Score*, and no two timed positions in the *Score* are assigned to the same timed position in the *Assignees*. The second condition enforces directionality within each individual assignment, i.e. it states that a position in the *Score* specified at time instant t_j must be assigned to a position in the *Assignees* specified at some time instant t_i earlier than t_j i.e. $t_i < t_j$. We call this the directionality constraint.

In addition to being *feasible*, if the total distance associated with the individual assignments in $\pi|_{As'}$ is minimum (akin to saying that the total distance traveled by all the robots is minimum), then we call it an *optimal assignment*, denoted by $\pi^\star$. Note that $\pi^\star$ is restricted to the subset $As' \in As$ because the condition $|As| \geq |Sc|$ forces $(|As| - |Sc|)$ number of timed positions in *As* to go unassigned, in order to ensure $\pi^\star$ is indeed a bijection.