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# Subspace exclusion zones for damage localization

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ABSTRACT

If a subdomain of a structural system is introduced to a perturbation, the resulting shifts in any field quantity outside the boundary of some closed region encompassing the perturbation can be generated from stress fields acting on the aforementioned boundary. In the present paper, this is exploited in the context of structural damage localization to cast the Subspace Exclusion Zone (SEZ) scheme, which locates damage by inspecting the feasibility of generating the observed shifts from actions acting at the boundary of the postulated zones in a model of the structure in question. As such, the SEZ scheme is a forward interrogation that allows for a user-defined localization resolution and, under certain input conditions, operates without the use of system identification. The approach is most conveniently implemented in the Laplace domain and holds at $s$-values for which the load vector in the reference and the damaged states are proportional. The constraint that ensures exact results in an idealized model context is that the number of measurements outside any considered exclusion zone (EZ) exceeds the number of DOF on its boundary. It is shown, however, that useful results can be obtained with notably smaller sensor counts. The paper illustrates application of the SEZ scheme in simulations and in an experimental setting using a beam subjected to harmonic input.



## 1. Introduction

When facing the task of locating a structural damage using vibration measurements, an approach often adopted is to map changes in some limited set of experimental features, for example, frequency response functions, to the structural domain by use of an analytical model [1–3]. The general premise is, as such, to compare analytical subspaces to an experimental subspace and then discriminate between healthy and damaged structural subdomains based on orthogonality (or parallelism) between these subspaces. One of the early examples of such approach is the Best Achievable Eigenvector technique [4], which is a forward method that interrogates one element at a time and announces damage when the span of a subspace that depends on the element being considered contains the identified eigenvectors. A shortcoming of this approach, and many other of the early localization schemes, is that the experimental feature needs to be available at all the coordinates of the analytical model. Since this requirement is never satisfied in practice, the missing experimental entries have to be estimated using coordinate expansion techniques [5], which often degrade the performance severely.

A damage localization approach that does not require a coordinate match between the experimental and the model coordinates is the Damage Locating Vector (DLV) family [6–9]. Here, damage is localized by inspecting stress fields computed in a

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model using load vectors from the kernel of changes in either the flexibility or the transfer matrix extracted from the measured vibration data; making evident that the approach relies heavily on sufficiently accurate system identification results. A localization scheme based on an extension of the subspace residual detection approach [10,11] is presented in [12]. This scheme, which also relies on system identification, is based on directional changes of a certain residual and is theoretically supported on the assumption that the damage is very small. Methods that attempt to localize damage by looking at changes in mode shapes have been examined by several authors [1,13], with "curvature mode shapes" receiving particular note in beam investigations. A review of the aggregate of the work up to year 2014 is presented in [14] and makes clear that the performance is generally poor. Another class of methods, restricted to cases where full control of the excitation sources is available, uses changes in transmissibility functions and a critical review can be found in [15]. There are, of course, many studies where damage is characterized by updating a model and information on location is obtained [16,17], but we do not consider these as damage localization techniques as the damage extent is not uncoupled in the formulation.

An item that often detracts from robustness in localization schemes is the fact that aspects of the problem that need not be answered affect results. For example, in a scheme that attempts to locate a crack in a plate, the crack orientation has an effect on the response but is of no interest as far as the localization goes. An approach that avoids the unnecessary complexity of damage details is the recently introduced Shaped Damage Locating Input Distribution (SDLID) scheme [18], which interrogates the domain from postulated damage patterns for which multiple controllable inputs are tailored to suppress certain vibration quantities. While circumvention of system identification and avoiding unnecessary details add merit to the SDLID scheme, it is obvious that not all applications allow the use of multiple controllable inputs. The approach introduced in this paper, the Subspace Exclusion Zone (SEZ) scheme, also avoids unnecessary damage details by interrogating the domain from the boundaries of closed regions postulated to contain the damage in their interior. The SEZ scheme, however, operates without the need for multiple controllable excitation sources and, under certain input conditions, free of system identification. The approach is built from the fact that a local perturbation-induced difference in any field quantity outside the boundary of some closed region—not necessarily simply connected—that contains the perturbation can be generated from stress fields acting on the aforementioned boundary. We note from the outset that the SEZ scheme does not require that the actual stress field on a boundary be determined. Instead, it operates solely with the question of whether it is possible, or not, to find such stress field. One gathers, from the previous discussion, that the SEZ approach does not directly point to the damage but rather allows one to test whether it is in the interior of any postulated exclusion zone (EZ).

While the SEZ scheme can be implemented either in the time or the Laplace (frequency) domain, the latter is the simpler choice and is, consequently, the one outlined throughout the present paper. The SEZ approach is most attractive in cases where the Laplace transform of the loading in the reference and the damaged states are proportional, because, in this instance, system identification is not needed. It is opportune to note that the proportionality requirement does not imply that the loading histories in the reference and the damaged states must be scaled versions of each other, but only that the Laplace transforms are proportional at the $s$-values used in the interrogation; a condition that is automatically satisfied for single-source loads. A noteworthy feature of the SEZ scheme is that the dimension of the EZs, and, consequently, the localization resolution, is user-defined and can be varied throughout the domain. In this way, a priori information on regions where damage is not feasible is easily considered. We note that in spite of the name selected for the scheme, whether or not the interior of the EZ is "removed" is immaterial; with the only difference being that in the "removed" case, the basis used to define the stress field at the "fictive boundary" is user-prescribed while, when the interior is not "removed", there is an unknown transformation which, as long as it is full rank, has no effect on results. This item, which plays a role in computational effectiveness, is further clarified in the body of the paper. The SEZ scheme bears strong resemblance to the recently introduced Steady State Shift Damage Localization (S3DL) scheme [19], which also operates in a forward mode with postulated analytical subspaces. The main difference between the two schemes is that the SEZ scheme rests on a theoretical premise allowing for structural interrogation without prior assumption of whether the damage is mass- or stiffness-related.

The SEZ premise is strictly constraining when the dimension of the experimental vector to be reconstructed is larger than the number of DOF at the EZ boundaries. One suspects, however, that the DOF at the boundary can be reduced by using a truncated basis dictated by the dominant singular values of the response fields generated from the boundary, and numerical results support this expectation. The remaining of the paper is organized as follows. First, in Section 2, the theoretical basis of the SEZ scheme is provided, followed by a brief summary of the scheme in Section 3. Section 4 contains the application examples while some concluding remarks are provided in Section 5.

## 2. The SEZ scheme

Consider a linear, time-invariant structural domain, $\mathcal{A}$, discretized with $n$ DOF and subjected to $e$ independent inputs gathered in $f(t) \in \mathbb{R}^e$ and distributed to $\mathcal{A}$ by $b_2 \in \mathbb{R}^{n\times e}$. For zero initial conditions, one has

$$x(s) = \left(Ms^2 + Cs + K\right)^{-1} b_2 f(s) = H(s) b_2 f(s), \tag{1}$$

where $M, C, K \in \mathbb{R}^{n\times n}$ are the mass, damping, and stiffness matrices, $x(s) \in \mathbb{C}^n$ is the nodal displacement vector, and $H(s) \in \mathbb{C}^{n\times n}$ is the receptance matrix. Now, if the structural domain is introduced to a change, and we assume that at the selected $s$-value, the Laplace transform of the loading is proportional to that in the reference state, one can write