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On the target frequency for harvesting energy from track vibrations due to passing trains

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ABSTRACT

There is an increasing desire to monitor the structural integrity of railway tracks and the supporting ballast and subgrade. Track vibration has been proposed as a potential energy source to power wireless sensors for this purpose. Vibration-based energy harvesting devices generally exploit resonance, and hence need to be tuned to a particular target frequency. Thus, to harvest energy from the vibrations of a passing train, the spectral content of the track vibration needs to be known. This paper describes a fundamental investigation into the factors that govern this spectral content. A simple model of the train and the track, together with data from five trains passing at four different sites are used in this investigation. It is shown that the deflection under an individual wheel effectively acts as a bandpass filter, restricting the acceleration spectrum of sleeper vibration to low frequencies. The train geometry has an important effect on which specific trainload frequency has the largest response amplitude. For the trains studied, it was found that the 7th trainload frequency had the highest amplitude in four out of the five cases. The physical reasons as to why this trainload frequency is the largest are discussed.

1. Introduction

The dominant source of low frequency vibration of a railway track is the quasi-static loading due to passing trains [1–3]. This excitation mechanism is characterized by the passage of the axle sequence of a train [4], resulting in large levels of vibration at the so-called trainload frequencies [5]. Between some of these frequencies, very low levels of excitation are present, and Krylov and Ferguson [6] have investigated this aspect with the aim of suppressing ground vibration at certain frequencies.

Track vibration can also be used to beneficial effect. Energy can be harvested from this vibration to power sensors that monitor the structural integrity of a railway track and its supporting ballast and subgrade. Nelson et al. [7] were one of the first research groups to study this problem using a piezoelectric transducer to harvest energy from the strain due to rail deflection. Several devices have also been proposed for the purpose of harvesting such energy. Li et al. [8] and Gao et al. [9] also used piezoelectric energy harvesters in the frequency range of 5 Hz to 7 Hz, and Wang et al. [10], Pourghodrat et al. [11] and Zhang et al. [12] used rotational generators to harvest energy in the frequency range of 1 Hz to 4 Hz. Electromagnetic

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energy harvesters have also been used to scavenge energy from the rail foot deflection in the frequency range of 3 Hz to 7 Hz [13].

However, there is a limited amount of work on which frequency of track vibration to target for energy harvesting, and the physical reasons for this. The aim of this paper is to fill this gap in knowledge. Some of the authors of this paper, recently carried out a fundamental investigation into the harvesting energy from track vibrations [14,15]. They found that the optimum target frequency was the 7th trainload frequency for the case that they studied. This was the frequency at which the vertical track acceleration was the largest. The question that remains unanswered, however, is whether the 7th trainload frequency is the frequency to target for all trains, and why this trainload frequency had the largest amplitude. To investigate this, a model is needed, together with some more data from other trains/sites. Two recent papers have helped in this regard. Le Pen et al. [16] presented a simple model of quasi-static excitation of the track from a train and used this model together with track velocity measurements to estimate the track support stiffness. Milne et al. [17] further developed this work, carrying out an investigation into how the shape of the velocity spectrum of the vertical track vibration is affected by the train and track properties. In this paper, acceleration, rather than velocity, is of interest, and five data sets (from different trains and sites) are studied. A simple model is used to interpret the data in the time and frequency domains. It is also used to determine the factors that govern the maximum vertical acceleration of track vibration and the frequency at which this occurs.

2. Model of sleeper vibrations due to a passing train

A drawing showing the cross-section of a typical rail track is shown in Fig. 1(a). The system consists of two steel rails connected to concrete sleepers by clips and rail pads. The sleepers rest on ballast and subgrade, which form the distributed track-bed stiffness per unit length of track, k_{trackbed}. The low frequency (<30 Hz) vertical vibration of the track is of interest in this paper. In this frequency range, the track vibration is predominantly that due to the quasi-static moving axle load. This is independent of the vehicle and track dynamics, whereas the track vibration due to dynamic excitation caused by the track and wheel irregularities is only significant at higher frequencies [18]. The model therefore neglects dynamic effects and uses a quasi-static model of the track. Each wheel of the train can be assumed to act as a steady point force on the beam moving at the train speed, S.

The first resonance of the track, corresponding to the mass of the rail and sleeper, interacting with the ballast stiffness, occurs around 37 Hz for the Steventon site rail track,[1] which is one of the tracks considered in this paper. Below this frequency, the track behaves simply as a stiffness. The rail is modelled as a beam on an elastic foundation. It is sufficient to use Euler-Bernoulli beam theory for the rail as higher order effects such as shear deformation are only significant above about 500 Hz [21]. As the distance between the sleepers, L_{sleeper}, (which is typically 0.65 m and is given in Table 1) is much smaller than the flexural wavelength in the rail at these low frequencies, the rail can be considered to be resting on an equivalent Winkler foundation with distributed support stiffness, $k_{\text{support}} = (L_{\text{sleeper}}/k_{\text{pad}} + 1/k_{\text{trackbed}})^{-1}$, in which k_{pad} is the stiffness of a rail pad, and the mass of the sleepers is ignored. Note that the pad is described as a point stiffness and the trackbed as a distributed stiffness, and that they combine in series to give the distributed support stiffness.

An example of a single wheel load on the model of a single rail system is shown in Fig. 1(b), where EI is the flexural stiffness of the rail, $w(x,t)$ is the vertical rail deflection, F is the moving load due to the wheel, δ is the delta function, and x is the horizontal position of the wheel, with reference to an arbitrary reference point. Fig. 1(c) shows a typical single carriage of a train, in which L_{car} is the length of the carriage, L_{wheel} is the distance between two wheels on a bogie, and L_{bogie} is the distance between the central points of the two bogies. The equation of motion for the rail with a single carriage acting on it is given by [22]

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + k_{\text{support}}\, w(x,t) = -\sum_{i=1}^{4} F_i \delta(St - x - d_i) \tag{1}$$

where t is time, F_i is the force due to the i-th wheel, and $d_1 = 0$, $d_2 = L_{\text{wheel}}$, $d_3 = L_{\text{bogie}}$ and $d_4 = L_{\text{wheel}} + L_{\text{bogie}}$. Noting that the relationship between the vertical displacements of the sleeper, w_{sleeper}, and the rail is given by $w_{\text{sleeper}}/w = k_{\text{support}}/k_{\text{trackbed}}$, the solution to Eq. (1) can be written

$$w_{\text{sleeper}}(x,t) = \frac{-\beta}{2k_{\text{trackbed}}}\sum_{i=1}^{4} F_i e^{-\beta|St - x_i - d_i|}[\cos(\beta(St - x_i - d_i)) + \sin(\beta|St - x_i - d_i|)] \tag{2}$$

where $\beta = (k_{\text{support}}/(4EI))^{1/4}$ has units of m^{-1}. Eq. (2) can be written in non-dimensional form as

$$\hat{w}_{\text{sleeper}}(\hat{x},\hat{t}) = \frac{w_s(x,t)}{\sum_{i=1}^{4} F_i/(2k_{\text{support}} L_{\text{car}})} = -\alpha\sum_{i=1}^{4} e^{-\alpha|\hat{t} - \hat{x} - d_i|}[\cos(\alpha(\hat{t} - \hat{x} - d_i)) + \sin(\alpha|\hat{t} - \hat{x} - d_i|)] \tag{3}$$

[1] The first resonance of the track, corresponding to the mass of the rail and sleeper interacting with the support stiffness, is given by [19] $\omega_1 = \sqrt{k_{\text{trackbed}}/(m_{\text{rail}} + m_{\text{sleeper}})}$, where m_{rail} and m_{sleeper} are the rail and sleeper masses per unit length, respectively. For the Steventon site $m_{\text{rail}} = 60$ kg/m; $m_{\text{sleeper}} = 245$ kg/m; and $k_{\text{trackbed}} = 16.5\ \text{MN/m}^2$ [20].