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A method for non-parametric identification of non-linear vibration systems with asymmetric restoring forces from a resonant decay response

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ABSTRACT

A method for non-parametric identification of systems with asymmetric non-linear restoring forces is proposed in this paper. The method, named the zero-crossing method for systems with asymmetric restoring forces (ZCA), is an extension of zero-crossing methods and allows identification of backbones, damping curves and restoring elastic and dissipative forces from a resonant decay response. The validity of the proposed method is firstly demonstrated on three simulated resonant decay responses of the systems with off-centre clearance, bilinear and quadratic stiffness. Then, the method is applied to experimental data from a micro-electro-mechanical resonator in order to quantify its non-linear damping and stiffness effects. Throughout the paper the proposed method is also compared with the Hilbert vibration decomposition to demonstrate that the ZCA yields more accurate results with much less effort.

1. Introduction

As non-linear behaviour becomes increasingly important in structural dynamics, so does an urgent need for experimental identification of non-linear systems. Numerous methods for non-linear system identification have been proposed [1–3]. A considerable number of studies has focused on non-linear system identification via time-frequency analysis, including the short-time Fourier transform [4] and wavelet transform [5]. More recently, methods based on instantaneous frequency (IF) and amplitude (IA), such as the Hilbert-Huang transform [6,7] and Hilbert transform [8,9], have been developed and used in structural dynamics for parametric and non-parametric identification [10–12]. To further improve the performance of these methods, some alternative methods for the estimation of the IF and IA, including zero-crossing methods [13,14], have been introduced.

A vast majority of non-linear system identification methods is limited to systems with symmetric restoring forces. Little research has been conducted on the problem of non-parametric identification of asymmetric restoring forces. The measured signals may however exhibit asymmetry with respect to time axis caused by different stiffness or dissipative forces for the positive and negative part of the motion. Such signals can originate from systems with bilinear, piecewise or off-set stiffness, which are not uncommon in engineering structures. For instance, gaps, end stops and pre-stress effects can exhibit asymmetric restoring forces.

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Nomenclature

Symbol	
a, f	instantaneous amplitude and frequency
f_{nl}	non-linear restoring force
$f_{\mathrm{u}}, a_{\mathrm{u}}$	frequency and amplitude from the upper part of a signal
$f_{\mathrm{l}}, a_{\mathrm{l}}$	frequency and amplitude from the lower part of a signal
m, c, k	mass, damping, stiffness of a linear system
$k_{\mathrm{nl}}, c_{\mathrm{nl}}$	quadratic stiffness and non-linear damping coefficient of a MEMS
k_1, k_2	bilinear stiffness coefficients
$t_i, t_{\mathrm{u}}, t_{\mathrm{l}}$	times of zero-crossings, maxima, and minima
$x, \dot{x}, \ddot{x}$	displacement, velocity, and acceleration
x_1, x_2	thresholds of off-centre clearance
y	smoothed displacement
$\mathrm{F_{el}}, \mathrm{F_d}$	elastic and dissipative restoring force
β, γ	off-centre clearance and quadratic stiffness coefficients
$\delta_{\mathrm{u}}, \delta_{\mathrm{l}}$	damping rates estimated from the upper and lower part of a signal
λ	smoothing parameter
HT	Hilbert transform
HVD	Hilbert vibration decomposition
IF, IA	instantaneous frequency and amplitude
SDOF	single-degree-of-freedom
MDOF	multi-degree-of-freedom
SNR	signal-to-noise ratio
ZC	zero-crossing method
ZCA	zero-crossing method for systems with asymmetric restoring forces

The Hilbert vibration decomposition (HVD) [8,9,15–17] is to the authors' knowledge the only time-frequency analysis method able to cope with a lack of symmetry in the restoring forces. The HVD for systems with asymmetric restoring forces is loosely based on the idea that each signal branch (the lower and upper part of the signal with respect to time axis) is defined on its half-plane only. So practically it is enough to identify matching instantaneous characteristics of each signal branch [16]. The HVD as used in [16] can determine not only the system's backbone and damping curves, but also the initial asymmetric non-linear elastic and dissipative restoring forces. However, the HVD involves extensive signal processing, may be sensitive to measured noise, and may suffer from numerical issues of the Hilbert transform. The HVD will be used as a reference method throughout the paper and it is therefore described in more details in Appendix A.

This paper proposes a new method, which avoids some of the problems experienced with the HVD, but which still allows non-parameter identification of the same class of systems. As the proposed method does not use the Hilbert transform it does not require sophisticated signal processing, it is easy to implement and is less sensitive to measured noise. The method is underlined by the same idea as the HVD for identification of asymmetric systems, i.e. the instantaneous characteristics of each signal branch are identified separately. Therefore, it provides equivalent results in terms of backbones, damping curves, elastic and dissipative restoring forces, while not requiring sophisticated signal processing. The method proposed identifies the instantaneous characteristics directly using modified zero-crossing approach from a resonant decay response and is named zero-crossing method for systems with asymmetric restoring forces (ZCA). Throughout the paper several sets of numerically and experimentally acquired data are used to validate the ZCA and a detailed comparison with the HVD is also made.

The paper is organised as follows: the zero-crossing method for asymmetric systems is proposed in Section 2. In Section 3 the ZCA is applied to three simulated cases, namely to the systems with bilinear stiffness, off-centre clearance (backlash) and quadratic stiffness. Section 4 then shows the application of the proposed ZCA as well as HVD to the data obtained experimentally from a micro-electro-mechanical beam. Section 5 discusses limitations and a range of applicability of the proposed method. The paper is complemented by two appendixes - a detailed description of the HVD is given in Appendix A and the Whittaker smoother, which is used in the paper to process noisy data, is described in Appendix B.

2. A zero-crossing method for systems with asymmetric restoring forces

This section introduces zero-crossing methods and subsequently the zero-crossing method for systems with asymmetric restoring forces is proposed. Both methods are applicable to a resonant decay response, so the way how such a response can be obtained is firstly described.