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Finite element model updating using objective-consistent sensitivity-based parameter clustering and Bayesian regularization

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ABSTRACT

Finite element model updating seeks to modify a structural model to reduce discrepancies between predicted and measured data, often from vibration studies. An updated model provides more accurate prediction of structural behavior in future analyses. Sensitivity-based parameter clustering and regularization are two techniques used to improve model updating solutions, particularly for high-dimensional parameter spaces and ill-posed updating problems. In this paper, a novel parameter clustering scheme is proposed which considers the structure of the objective function to facilitate simultaneous updating of disparate data, such as natural frequencies and mode shapes. Levenberg–Marquardt minimization with Bayesian regularization is also implemented, providing an optimal regularized solution and insight into parametrization efficiency. In a small-scale updating example with simulated data, the proposed clustering scheme is shown to provide moderate to excellent improvement over existing parameter clustering methods, depending on the accuracy of initial model. A full-scale updating example on a large suspension bridge shows similar improvement using the proposed parametrization scheme.

1. Introduction

Modern structural analysis generally depends on finite element (FE) models to predict dynamic behavior and understand the current state of a system. Though these models are often developed from detailed design drawings, discrepancies always exist between measured (observed) and model-output behavior [1]. Typical sources of discrepancy are model idealization, FE discretization errors, and uncertain model parameters such as material properties, section properties, geometry, and boundary conditions [1,2]. Discrepancies indicate that a model cannot reliably predict the behavior of its corresponding physical structure, limiting the utility of the model for future analysis.

Model updating is the process which seeks to reduce discrepancies between measured data and model-output data by adjusting parameters of an FE model [1–3]. Model updating has been successfully applied to a wide variety of aerospace, mechanical, and civil structures. Examples include a helicopter airframe [2,4], an aluminum space-frame [5], a prestressed

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single-span highway bridge [6], a prestressed multi-span highway bridge [7], a concrete-filled steel tubular arch bridge [8], an actively-damped high-rise structure [9], and a residential reinforced concrete frame [10].

Model updating techniques may be divided into two categories: uncertainty quantification (UQ) methods and deterministic methods [11]. UQ methods incorporate measurement and model uncertainties in their solutions and can be grouped into probabilistic and non-probabilistic approaches. Probabilistic UQ methods estimate probability distributions functions for parameters and model outputs through repeated sampling in the parameter space. The most common non-probabilistic UQ method is fuzzy model updating, which uses optimization to estimate intervals for parameters and outputs corresponding to upper and lower bounds of measured data. However, both probabilistic and non-probabilistic UQ methods are orders-of-magnitude more computationally expensive than deterministic methods. An excellent review of UQ model updating can be found in [11].

Deterministic model updating produces a unique optimal solution and typically involves iterative adjustment of FE model parameters [3]. Of course, as these schemes generally involve minimizing a non-linear function, they are possibly subject to convergence problems. Among iterative methods, the sensitivity method [2] is one of the most intuitive and popular techniques for model updating. The sensitivity method approaches model updating as a non-linear least-squares minimization problem which is solved by iterations of linear approximations. The objective function is a sum of squared differences between measured and model-output data, making it easy to incorporate various data. The use of linear approximations also makes this method physically-intuitive and efficient, as the Jacobian matrix is directly relatable to model parameter sensitivities. However, the sensitivity method is often applied to ill-posed model updating problems, necessitating a reduction in the number of updating parameters and/or the inclusion of side-constraints in order to reach a unique, stable solution.

Shahverdi et al. [4] presented sensitivity-based parameter clustering as a viable method for reducing the number of updating parameters. By observing the sensitivities of model outputs to changes in model parameters, sensitivity-based parameter clustering generates clusters of model parameters which have similar effects on targeted model outputs. Then, each cluster of model parameters is updated by a single parameter. This gives a reduced-order model, generally with a better-conditioned Jacobian, while retaining the physical relevance of clustered model parameters. This technique was successfully applied to the updating of a helicopter airframe [4]. Jang and Smyth [12,13] applied this method for the updating of a large-scale suspension bridge.

Regularization is another technique used to solve ill-posed and noisy problems which often occur in FE model updating [2,14–16]. Generally, regularization adds equations which help constrain the updating solution. This can help produce a unique solution to an underdetermined problem (fewer measurements than parameters), though this situation should be avoided. Regularization is often used to give a minimum-norm solution, but it may also be used to enforce user-specified constraints between parameters [2].

While sensitivity-based parameter clustering is very promising, it is difficult to utilize disparate sources of data, such as natural frequencies and mode shapes, due to differences in scale. Previous work with parameter clustering only used one type of data [2,4], or used only natural frequency sensitivities for clustering despite the inclusion of mode shapes in the objective function [12,13]. To alleviate scaling issues during parameter clustering, it is necessary to develop a weighting technique which is efficient and reflective of the problem structure. The presented research details an objective-consistent weighting technique based on the residual. This paper also implements Bayesian regularization [17,18] in model updating, which gives a statistically optimal regularized solution. Bayesian regularization also provides insight into the effective number of updating parameters, which is used to explore the efficiency of competing parametrizations.

The paper begins with the definition of residual between measurements and corresponding model outputs, along with analytical sensitivities of model outputs to model parameters (Section 2). Model parametrization, clustering, and the objective-consistent weighting scheme are discussed in Section 3. The Levenberg–Marquardt minimization method, with the accompanying Bayesian regularization technique, are detailed in Section 4. Two model updating exercises are then performed to exhibit the efficiency of the objective-consistent clustering scheme for simultaneous updating of natural frequency and mode shape data. The first exercise uses a small-scale 2-dimensional truss with simulated measurements (Section 5), while the second uses a full-scale large suspension bridge with real data (Section 6). The findings are then discussed and concluding remarks are made in Section 7.

2. Residual definition and analytical sensitivity of model parameters

The sensitivity method for FE model updating [2] begins with the definition of a discrepancy, or residual, to be minimized by modifying a set of updating parameters. Traditionally, the residual $\boldsymbol{r}$ is defined as the difference between the column vector of m measured outputs $\tilde{\boldsymbol{z}}$ and the column vector of m analytical model outputs $\boldsymbol{z}(\boldsymbol{\theta})$ which is a function of the p updating parameters $\boldsymbol{\theta}$. The relationship between $\boldsymbol{r}$ and $\boldsymbol{\theta}$ is generally non-linear, but can be linearized by truncating the Taylor series after the linear term:

$$\boldsymbol{r}(\boldsymbol{\theta}) = \tilde{\boldsymbol{z}} - \boldsymbol{z}(\boldsymbol{\theta}) \approx \boldsymbol{r}(\boldsymbol{\theta}_i) + \boldsymbol{J}_i(\boldsymbol{\theta} - \boldsymbol{\theta}_i) \tag{1}$$

At iteration i, $\boldsymbol{\theta}_i$ is the updating parameter vector and $\boldsymbol{J}_i \in \mathbb{R}^{m \times p}$ is the Jacobian matrix of $\boldsymbol{r}$ with respect to $\boldsymbol{\theta}$, evaluated at $\boldsymbol{\theta}_i$:

$$\boldsymbol{J}_i = \left.\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{\theta}}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_i} \tag{2}$$