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Synchro-squeezed adaptive wavelet transform with optimum parameters for arbitrary time series

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ABSTRACT

In this study, a synchro-squeezed adaptive wavelet transform (SSAWT) is proposed and defined as an average of overlapped short-time wavelet transforms with optimized time-varying resolution in a synchro-squeezed time-frequency representation. The time-frequency resolution is automatically updated with a simplified procedure to determine optimal wavelet parameters. The instantaneous frequency spectra are accumulated over time to extract time-insensitive frequency characteristics in arbitrary time series. An illustrative signal with four time segments covering various frequency distribution cases indicated a 1.1% error of the proposed method, which is at least 5 times more accurate than the conventional synchro-squeezed wavelet transform. Due to synchro-squeezing process, SSAWT also exhibited more accurate results than the adaptive wavelet transform that has recently been developed by the authors. The proposed SSAWT was then applied to the impact echo responses experimentally recorded from a 60″ × 36″ × 7.25″ concrete slab. The improvement in time-frequency resolution and corresponding frequency spectra led to more successful detections of deep or shallow or no delamination from 40 sets of experimental data within 1.5% estimation error in deep delamination depth and 5% estimation error in slab thickness.

1. Introduction

Several signal processing techniques have been developed to represent a time-varying signal in frequency domain or in time-frequency domain. The most widely used technique is Fourier transform (FT) [1], which simply transforms a signal from time to frequency domain. FT spectra retain no time information and make it impossible to identify time-varying frequencies that are commonly encountered in civil engineering structures under extreme events or with energy dissipation devices. Short-time Fourier transform (STFT) [2] was thus developed by taking the FT of many segments of a signal over non-overlapping time ranges, giving a time-frequency representation (TFR) of the entire signal. However, several trials are often required to capture signal characteristics due to fixed frequency resolution over the entire duration of the signal.

Therefore, continuous wavelet transform (CWT) was developed to provide variable frequency resolution at each time by introducing scaling and shifting factors to a mother wavelet [3]. The same range of frequency resolution at every time allows an effective decomposition of complicated signals in engineering applications [4–6]. For example, a CWT filter was used to analyze the signal of a gearbox for fault diagnosis according to the kurtosis maximization principle [7]. CWT was also used to determine the threshold in denoising algorithm [8].The compression method with an optimum threshold in CWT was

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developed for image denoising [9]. CWT and the least-mean-square (LMS) algorithm were applied together for the determination of filter parameters [10]. A CWT-based method was even proposed for electroencephalogram identification [11]. CWT was also used to determine bisecting frequencies for the decomposition of stationary, non-stationary and nonlinear signals [12,13]. Use of CWT for impact echo (IE) responses can also be found in several applications of PVC tubes [14] and concrete slabs [15–18]. Approaches of optimizing CWT have been proposed by taking advantage of neural network [19–21], genetic algorithm [22]. All of the above studies analyzed either the entire signal or completely separated individual segments.

Although CWT is desirable in several ways, it still gives dispersive and blurry scalograms/ridgelines in time-frequency domain. As such, synchro-squeezed wavelet transform (SSWT) [23] was recently developed to acquire a more accurate TFR with sharpened ridgelines. However, both CWT and SSWT are unable to change frequency resolution over time. A few attempts with adaptive signal representation techniques such as matching pursuit [24], best basis [25] and basis pursuit [26] were based on discrete TFRs for optimal information encoding and compression instead of the extraction of signal characteristics. Therefore, adaptive wavelet transform (AWT) was recently proposed to improve the frequency resolution over time [27]. Even then, the resulting scalogram in time-frequency domain consists of lines with finite width, which can be further refined for the identification of characteristic frequencies.

This study represents the first report on the synchro-squeezed short-time adaptive wavelet transform (SSAWT). The SSAWT is first defined as an average of synchro-squeezed short-time CWT segments that are staggered over time. The wavelet parameters for each time segment of an arbitrary signal are then optimized with a new algorithm to be developed. Next, the characteristics and performance of the proposed SSAWT are evaluated using an analytical signal of two closely-spaced frequency-modulated sinusoids. Finally, the proposed SSAWT is applied to the IE responses experimentally recorded from a concrete slab to identify its thickness or detect embedded delamination defects. The identified results are compared with those by the conventional CWT, SSWT and the recently developed AWT.

2. Synchro-squeezed adaptive wavelet transform algorithm

2.1. Definition and formulation

Synchro-squeezing is a special case of reallocation methods. When integrated into the reversible SSWT, it can help sharpen the scalogram/ridgelines obtained from the CWT [28]. Before SSAWT is developed, the SSWT is briefly reviewed. The CWT represents the convolution of a signal, $x(t)$, and the conjugate of an appropriately chosen scaled mother wavelet, ψ:

$$W_x(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t)\overline{\psi_{a,b}(t)}dt, \tag{1}$$

where $W_x(a,b)$ is the wavelet coefficient as illustrated in Fig. 1(a) (module of the wavelet coefficient if it is a complex function) when $x(t) = cos(2\pi 8t)$. It represents how similar the signal is to the mother wavelet when shifted by a certain time b and scaled by a factor of a. The wavelet coefficient is synchro-squeezed and transferred from the time-scale plane to a time-frequency plane by:

$$SSWT\{x(t)\}(\omega,b) = \int_{A(b)} W_x(a,b)a^{-3/2}\delta(\omega_x(a,b)-\omega)da \tag{2}$$

where $A(b) = \{a : W_x(a,b) \neq 0\}$, and $\omega_x(a,b) = -i\frac{1}{W_x(a,b)}\frac{\partial[W_x(a,b)]}{\partial b}$. The synchro-squeezed transform corresponding to Fig. 1(a) is presented in Fig. 1(b) (coefficient module). It is clearly seen from Fig. 1(b) that the frequency of the signal is 8 Hz.

The recently developed AWT is a signal processing method that adaptively adjusts itself to the time-varying signal by changing the center frequency of a Morlet mother wavelet in order to achieve the selectivity of time and frequency

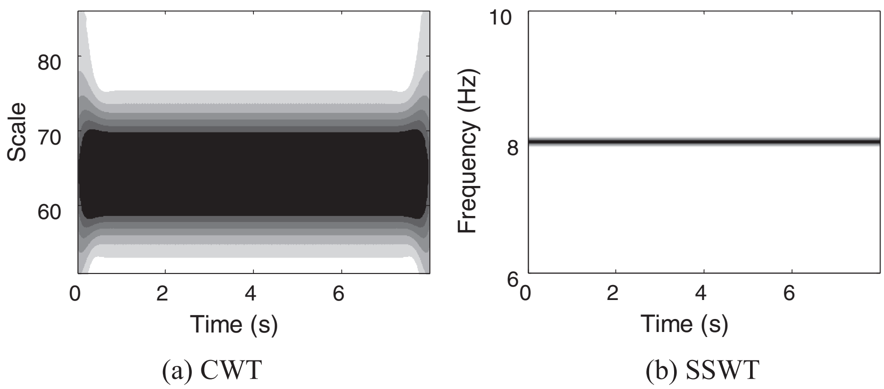

Fig. 1. SSWT illustration of $x(t) = \cos(2\pi 8t)$.