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Brief paper

# Consensus-based decentralized real-time identification of large-scale systems ☆

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ABSTRACT

In this paper an approach is proposed to decentralized multi-agent identification of large-scale systems represented by linear discrete-time stochastic MIMO models. It is assumed that each agent: (a) has access only to a subset of noisy input–output variables; (b) communicates local data processing results to its neighborhood. The proposed algorithm consists of two stages. The first stage is a consensus-based stochastic approximation algorithm for estimating input–output correlation functions, while at the second stage each agent utilizes a stochastic approximation algorithm with expanding truncations derived from the modified Yule–Walker equations in order to generate all the system parameter estimates. It is proved that under nonrestrictive assumptions concerning the system properties and the multi-agent network topology the estimates of the correlation functions converge almost surely to their true values and those of the system parameters to a solution of the modified Yule–Walker equations, assuming intermittent observations and communication outages. Conditions are also given for the strong consistency of the parameter estimates. Simulation results provide an illustration of the algorithm properties.



## 1. Introduction

Theory and practice of complex and large-scale systems have been focused during many years on decentralized methods, *e.g.*, Šiljak (1991). The attention paid recently to networked control systems, multi-agent systems and sensor networks have emphasized their importance, so that many efficient *decentralized methods for control and estimation* have been proposed and applied in practice, *e.g.*, Lunze, Heemels, and Schutter (2013) and references therein. Also, it has been found that methodologies based on *dynamic consensus* can represent a convenient tool for achieving a successful compromise between decentralization of functions and global system performance, *e.g.*, Tsitsiklis, Bertsekas, and Athans (1986), Kushner and Yin (1987), Olfati-Saber, Fax, and Murray (2007), Stanković, Stanković, and Stipanović (2009b), Stanković, Stanković, and Stipanović (2009a), Stanković, Stanković, and Stipanović (2011b), Chen and Sayed (2012), Bianchi, Fort, and Hachem (2012), Nedić and Olshevsky (2015), Stanković, Ilić, and Stanković (2014).

However, to the best of the authors knowledge *decentralized identification of dynamic large-scale systems* has not yet been considered in the literature from a general standpoint (see Irshad, Mossberg, & Söderström, 2013; Sim, Carbonnell-Marquez, Spencer, & Jo, 2011 for interesting specific approaches). The main idea of decentralized identification is to get a *global model* of a large-scale system by using *distributed measurements* performed by a sensor network in which each node has *access only to a subset of input and output variables* (*e.g.*, according to its location or functional properties) and possesses *limited computational and communication capabilities*. In this setting, the sensor network itself is introduced for monitoring and diagnostic purposes and does not influence dynamics of the system to be identified. Practical engineering motivations are numerous and can be found in diverse areas of engineering (see, *e.g.*, Nagayama, Sim, Miyamori, and Spencer, J (2007), Jo, Sim, Nagayama, and Spencer (2011), Lunze

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et al. (2013) and the references therein). More specifically, they include: system parameter estimation for monitoring and fault detection and isolation in large mechanical constructions (bridges, oil platforms, large space constructions) or large buildings, monitoring of the environment by flocks of mobile robots (the case of mobile sensor networks), monitoring of large water supply and distribution systems, system parameter estimation for monitoring and fault detection and isolation in a system of hydro power plants, monitoring and fault detection of furnaces and coal supply in large thermal power plants, estimation and fault detection in large production lines, large chemical plants, iron and steel industry, etc.

The main conceptual problem intrinsic to decentralized identification lies in the contradiction between the decentralized nature of data acquisition and local processing imposed by *information structure constraints*, on the one side, and the inherent interconnectedness between all the system variables preventing *ad hoc* system decomposition, on the other. The existing results related to identification with missing data are far from being able to provide any methodologically consistent general solution, especially for real time applications (*e.g.*, Aguero & Goodwin, 2008; Carvajal, Delgado, Aguero, & Goodwin, 2012; Little & Rubin, 2002).

In this paper a consistent approach to *decentralized real-time identification* of large-scale systems is proposed. Starting from a multi-agent setting, the method utilizes local data processing in real-time and inter-agent communications of the obtained results aimed at achieving a consensus and ensuring global character of the system model. As a final result, each agent may possess the estimates of all the system model parameters, *without recurring to any type of centralized decision or fusion center*.

Formally, it is assumed that the large-scale system to be identified is represented by a general linear MIMO (multiple input–multiple output) discrete time model with the input generated by a MIMO ARMA model (see, *e.g.*, Ljung, 1989, Chen, 2002 and Söderström & Stoica, 1989). An algorithm is proposed in which, at the first stage, the multi-agent sensor network is aimed at estimating the set of input–output *correlation functions* using locally available input–output measurements contaminated by noise and utilizing a dynamic consensus algorithm over the communication graph defined in accordance with the adopted communication structure constraints. In this sense, the methodology from, *e.g.*, Stanković et al. (2009a,b, 2011b) oriented at consensus-based decentralized parameter and state estimation is used to extend the idea presented in Chen (2007) to the multi-agent environment. At the second stage, the current correlation function estimates are used within stochastic approximation schemes with expanding truncations derived from the modified Yule–Walker equations to provide estimates of all the system model parameters to each agent, *e.g.*, Chen (2002, 2007); Söderström and Stoica (1989); Stoica (1983). One of the important contributions of the paper is a rigorous proof of almost sure (a.s.) convergence of the correlation function estimates to their true values under a set of nonrestrictive assumptions (methodologically distinct from the approaches in Bi-Qiang and Chen (2014); Chen (2007), related to a classical centralized scheme); it is also proved that the global model parameter estimates converge to a solution of the modified Yule–Walker equations. The strong identifiability issue is treated in a separate theorem. *Communication outages* and *intermittent measurement* are assumed with positive probabilities. Asynchronous functioning of the network in the presence of communication delays is also outlined. Simulation results provide an illustration of the performance of the proposed algorithm.

## 2. Problem definition and algorithm formulation

Let a dynamic discrete-time multiple input–multiple output (MIMO) system **S** be given, with the input vector $u(t) = [u_1(t)\cdots u_m(t)]^T$ and the output vector $y(t) = [y_1(t)\cdots y_n(t)]^T$, where $t$ denotes the discrete time instant; let $z(t) = [y(t)^T\ u(t)^T]^T$. Assume a situation in which $N$ autonomous agents measure inputs and outputs of **S** in such a way that each agent has access to a locally available subset of input/output variables contaminated by local additive white measurement noises. Let the index set $S^{(i)}$ contain the indices corresponding to the components of $z(t)$ accessible to the $i$th agent. In addition, we will assume that local measurements may not be available at certain time instants due to sensor faults (intermittent observations). Consequently, the *local measurement vectors* are defined by $z^{(i)}(t) = \mathrm{diag}\{I^{(i)}(t)\}\tilde{z}^{(i)}(t)$, where $\tilde{z}^{(i)}(t) = z(t) + \xi^{(i)}(t)$, $i = 1,\ldots,N$, $\xi^{(i)}(t) = [\xi_y^{(i)}(t)^T\ \xi_u^{(i)}(t)^T]^T$ is the local measurement noise $(n+m)$-vector (having formally the maximal number of components) and $I^{(i)}(t)$ is an $(n+m)$-vector of random binary elements such that $I_k^{(i)}(t) = 0$ a.s. for $k \notin S^{(i)}$, for every time instant $t$.

According to the above presented general setup, we assume that the system **S** is described by the following general model

$$A(q)y(t) = B(q)u(t), \qquad P(q)u(t) = Q(q)\varepsilon(t), \tag{1}$$

where $y(t)$ are measurable outputs, $u(t)$ are measurable inputs, $q$ is, depending on the context, either the backward shift operator or complex variable, $\varepsilon(t)$ is the unobservable input, while $A(q) = I + A_1q + \cdots + A_{\nu_a}q^{\nu_a}$, $B(q) = B_1q + \cdots + B_{\nu_b}q^{\nu_b}$, $P(q) = I + P_1q + \cdots + P_{\nu_p}q^{\nu_p}$ and $Q(q) = I + Q_1q + \cdots + Q_{\nu_q}q^{\nu_q}$, with $A_i = [a_{jk}^{[i]}]$, $i = 1,\ldots,\nu_a$, $j,k = 1,\ldots,n$, $B_i = [b_{jk}^{[i]}]$, $i = 1,\ldots,\nu_b$, $j = 1,\ldots,n$, $k = 1,\ldots,m$, $P_i = [p_{jk}^{[i]}]$, $i = 1,\ldots,\nu_p$, $j,k = 1,\ldots,m$ and $Q_i = [q_{jk}^{[i]}]$, $i = 1,\ldots,\nu_q$, $j,k = 1,\ldots,m$.

From (1) we have the compact form $G(q)z(t) = S(q)\varepsilon(t)$, where $G(q) = \begin{bmatrix} A(q) & -B(q) \\ 0 & P(q)\end{bmatrix} = I + G_1q + \cdots + G_\nu q^\nu$, $G_i = \begin{bmatrix} A_i & -B_i \\ 0 & P_i\end{bmatrix}$, $i = 1,\ldots,\nu$, $S(q) = \begin{bmatrix} 0 \\ Q(q)\end{bmatrix} = S_0 + S_1q + \cdots + S_\nu q^\nu$, $S_i = \begin{bmatrix} 0 \\ Q_i\end{bmatrix}$, $i = 1,\ldots,\nu$, where $\nu = \max(\nu_a,\nu_b,\nu_p,\nu_q)$, with $A_i = 0$, $B_j = 0$, $P_k = 0$, $Q_s = 0$ for $i > \nu_a$, $j > \nu_b$, $k > \nu_p$, $s > \nu_q$ (Chen, 2007; Ljung, 1989; Söderström & Stoica, 1989). Introducing $Z(t;\nu) = [z(t)^T \cdots z(t-\nu+1)^T]^T$ we obtain

$$z(t) = -GZ(t-1;\nu) + S(q)\varepsilon(t), \tag{2}$$

where $G = [G_1 \cdots G_\nu]$ is the parameter matrix.

The following assumptions concerning the system and its variables are adopted at this point:

(A.1) $\{\xi^{(i)}(t)\}$, $i = 1,\ldots,N$, and $\{\varepsilon(t)\}$ are sequences of i.i.d. zero mean random vectors independent from each other, with $E\{\xi^{(i)}(t)\xi^{(i)}(t)^T\} = R_\xi^{(i)}$ and $E\{\varepsilon(t)\varepsilon(t)^T\} = R_\varepsilon$, satisfying $E\{\|\xi^{(i)}(t)\|^{2+\gamma}\} < \infty$ and $E\{\|\varepsilon(t)\|^{2+\gamma}\} < \infty$ for some $\gamma > 0$.

(A.2) All the roots of $\det A(q)$ and $\det P(q)$ are outside the closed unit disk.

Under (A.2), $\{z(t)\}$ is a stationary and ergodic sequence with $E\{\|z(t)\|^2\} < \infty$, so that $E\{z(t)z(t-\tau)^T\} = R^{zz}(\tau) = [r_{ij}^{zz}(\tau)]$, $(i,j = 1,\ldots,m+n)$. Multiplying (2) by $Z(t-\nu-1;\mu)$ $(\mu \geq \nu)$ from the right and taking the mathematical expectation one obtains the *modified Yule–Walker equations*:

$$G\Gamma + W = 0, \tag{3}$$

where $W = E\{z(t)Z(t-\nu-1;\mu)^T\}$ and $\Gamma = E\{Z(t-1;\nu)Z(t-\nu-1;\mu)^T\}$ (Chen, 2007; Stoica, 1983). The solutions of (3) are given by $G = -W\Gamma^+ + G_0(I - \Gamma\Gamma^+)$, where $G_0$ is any matrix with appropriate dimensions. The necessary and sufficient condition for uniquely defining $G$ from (3) is non-degeneracy of $\Gamma$ ($A^+$ stands for the pseudoinverse of a matrix $A$) (Chen, 2007). Replacing $z(t)$ with its noisy version $\tilde{z}^{(i)}(t)$ in $Z(t;\nu)$, one obtains $\tilde{Z}^{(i)}(t;\nu) = [\tilde{z}^{(i)}(t)^T \cdots \tilde{z}^{(i)}(t-\nu+1)^T]^T$. It is important to notice that (A.1) implies that $E\{\tilde{z}^{(i)}(t)\tilde{Z}^{(i)}(t-\nu-1;\mu)^T\} = W$ and $E\{\tilde{Z}^{(i)}(t-$