Mechanical Systems and Signal Processing 112 (2018) 1–9

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Synchronization of two self-excited pendula: Influence of coupling structure's parameters

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ARTICLE INFO

Article history:
Received 11 December 2017
Received in revised form 22 March 2018
Accepted 15 April 2018

Keywords:
Mechanical systems
Co-existing attractors
Complicated basins
Energy balance

ABSTRACT

In this paper we investigate the possibilities and scenarios of synchronization under the influence of parameters of structure that couples oscillators. The research is based on a simple model of two pendula with van der Pol damping type, suspended on a horizontally oscillating beam. The regions of existence and co-existence of different synchronization patterns are shown, and typical bifurcation scenarios between types of model's responses are presented. The behavior of each pendulum is analyzed, and the reasons of bifurcations are explained using the energy balance method. Moreover, we investigate the basins of attraction of considered system, exhibiting their complicated structure along with statistical study on attractors' occurrence. Our results are presented for simple one parameter variation, as well as for group of parameters varied, when the influence of coupling structure's parameters on appearance of observed patterns is studied. Our results refer to a simple and typical mechanical model of coupled oscillators, emphasizing their universality in dynamical systems of engineering type.

1. Introduction

Synchronization is one of the most fundamental phenomenon occurring in nature and can be observed almost everywhere around us. With its scientific origins in the study on mechanical systems and famous Huygens experiments with clocks [1], it has been thoroughly investigated by many researchers concerning engineering problems, and generally, nonlinear dynamics [2–5]. Although it is known for years, many innovative works are continuously published, especially concerning complex models. One of the very first studies of synchronization of chaotic systems can be found in [6], while in [7,8] Authors analyze the synchronization of Lorenz systems with sliding mode control and pinning synchronization of complex networks, respectively. The study on control can be found in [9], while its avoidance has been discussed in [10]. Nowadays, research on synchronization can have a huge impact on modern problems (just to mention digital secure [11] or climate [12]), exhibiting that study of this phenomenon still has an enormous potential of discovering new and interesting results.

Analysis on synchronization is naturally linked with the area of engineering sciences, and in particular with one of the most fundamental models describing the motion occurring in nature, becoming the prototype of all mechanical systems, i.e. the pendulum. The starting point constituted by Huygens in 1665 [1] expanded into a large series of scientific studies

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https://doi.org/10.1016/j.ymssp.2018.04.025
0888-3270/

on possibilities of pendula synchronization, which results allow us to better understand the dynamical behaviors in mechanics.

Synchronization of pendula has been observed in variety of models, just to mention self-excited ones suspended on the displacing beam [13], or chains of nonlinear systems connected by linear springs [14]. The possibilities of clustering of units have been analyzed in [15], while in [16] Authors study transitions into complete synchronization for two diffusively coupled chaotic oscillators with parametric excitation. In [17] Perlikowski et al. presented the chaotic patterns of coupled clocks. Study on multistability and state estimation of pendula systems over digital communication channels can be found in [18,19], respectively. The reasons for synchronization and loss of stability have been described in [20,21], where the study on energy balance of considered systems is shown. Apart from typical phenomena, pendula models can exhibit also more complex and unpredictable dynamics, e.g. one of the most studied nowadays chimera states [22,23].

This paper continues research of previous works [17,20,21], representing however a novel approach to investigated model, which has not been considered earlier. In contrast to studies shown in [17,20,21], which have been performed for unchanged properties of coupling structure (beam), here we vary its parameters, showing their essential relevance in the possibility of observation of desired patterns. Our motivation is to evaluate how the oscillators' dynamics depends on the typical characteristics describing structure that couples the units, and to determine the regions where different types of their synchronization is possible.

Our results in this paper are ordered as follows. Our model is introduced in Section 2. In Section 3 we investigate the effect of the variation of the structure's mass on the behavior of coupled pendula. Then, in Section 4 we vary a group of characteristics, uncovering the influence of beam's suspension parameters on possible dynamics. Works performed in this study are concluded in Section 5 with remarks and possible implications of presented results.

2. Model

Our research is based on a simple model of two self-excited pendula, coupled through a horizontally moving beam [17,20,21]. The scheme of investigated system is shown in Fig. 1, while its dynamics based on the 2nd order Lagrange equations is given as follows:

$$\begin{cases} (M+m_1+m_2)\ddot{x} + m_1 l_1(\ddot{\varphi}_1\cos\varphi_1 - \dot{\varphi}_1^2\sin\varphi_1) \\ \quad + m_2 l_2(\ddot{\varphi}_2\cos\varphi_2 - \dot{\varphi}_2^2\sin\varphi_2) + c\dot{x} + kx = 0, \\ m_1 l_1\ddot{x}\cos\varphi_1 + m_1 l_1^2\ddot{\varphi}_1 + c_\varphi(\mu\varphi_1^2 - 1)\dot{\varphi}_1 \\ \quad + m_1 g l_1\sin\varphi_1 = 0, \\ m_2 l_2\ddot{x}\cos\varphi_2 + m_2 l_2^2\ddot{\varphi}_2 + c_\varphi(\mu\varphi_2^2 - 1)\dot{\varphi}_2 \\ \quad + m_2 g l_2\sin\varphi_2 = 0. \end{cases} \tag{1}$$

The dynamical variables x and $\varphi_1, \varphi_2 \in (-\pi, \pi]$ denote position of the beam and the angular displacement of 1st and 2nd pendulum, respectively. The beam of mass M [kg] is connected with the support by spring of stiffness k [N/m] and damper of damping coefficient c [Ns/m]. On the other hand, m_i [kg] and l_i [m] denote the mass and length of i-th pendulum, where $i = 1, 2$. Moreover, each pendulum is damped by the van der Pol type nonlinearity [24,25] (not shown in Fig. 1), with parameters c_φ and μ. Due to the nonlinear damping, system (1) is self-excited and the energy can be transferred from one pendulum to another through the moving beam.

In order to solve Eq. (1), we have used the 4th order classical Runge-Kutta method. The time step in our numerical study has been fixed at $h = \frac{2\pi}{N}\sqrt{\frac{l_2}{g}}$, where accuracy equals $N = 1000$, and each bifurcation step has been calculated for 1500 iterations of 2nd pendulum's natural period, i.e. $1500hN$ [s]. Results presented in this research have been performed in original Authors' programs, using C++ language.

3. Influence of beam's mass

In this section we have fixed some of the parameters to uncover possible system's responses when one of the beam's parameter, namely beam's mass M is varied. For this case, the stiffness and damping coefficient are fixed at $k = 4.0, c = 1.53$, while $M \in (0, 2]$ is considered as the bifurcation parameter. For considered set of values beam is always

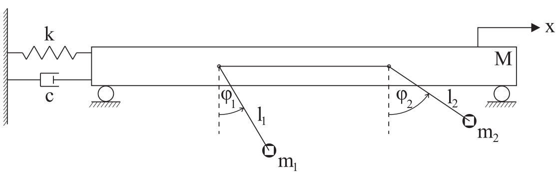

Fig. 1. Scheme of investigated model of two pendula suspended on a beam.