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Output feedback in the design of eigenstructures for enhanced sensitivity

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ABSTRACT

The problem of closed-loop enhanced sensitivity design is as follows: given a linear, time-invariant system, find a (realizable) feedback gain such that (1) the closed-loop is stable in the reference and the potentially damaged states, and (2) the eigenstructure includes a subset of poles, with desirable derivatives, that lie in a part of the plane where identification is feasible. For state feedback the eigenstructure is typically assignable and stability in the reference state is easily enforced. For output feedback, however, only partial assignment is possible, and it is here shown that the standard SVD design scheme leads to generically unstable eigenstructures when measurands are homogeneous (that is, when all sensors measure displacements, velocities, or accelerations). The mechanics that govern this behavior are clarified and a mitigating strategy that retains the convenience of homogeneous sensing is offered.

1. Introduction

Eigenstructure assignment is a control design scheme where objectives are attained by directly specifying closed-loop poles and eigenvectors [1]. The topic of this paper is eigenstructure assignment using output feedback when the purpose of the assignment is the realization of pole sensitivities favorable for Structural Health Monitoring (SHM) purposes [2–4]. A difficulty in designing for this objective derives from the fact that output feedback allows only partial control of the eigenstructure and that little can be said about where the poles that are not directly assigned end up [5]. The foregoing would not be an important issue if instability was encountered sporadically in the search for suitable gain, but results show that this is not so. Instead, what is found is that in the common case where measurands are homogeneous, that is, when all sensors are of the same type, instability in the optimization search is the norm, not the exception.

The reason why homogeneous sensing enters the problem is best appreciated in the derivations, but it can be outlined qualitatively from the outset and we do so next. Namely, in the typical (and most convenient) parameterization the gain is computed as the product of two matrices where one is the inverse of a matrix, $\boldsymbol{\Psi}$, that lists, at the measured coordinates, the right-side eigenvectors of the placed poles of the closed-loop transition matrix. The eigenvectors of the first order formulation can be written as $\psi_j = \left\{ \boldsymbol{\varphi}_j \quad \boldsymbol{\varphi}_j \lambda_j \right\}^T$, where $\boldsymbol{\varphi}_j$ is the latent vector of the second-order formulation, and since the gain is real, the columns of $\boldsymbol{\Psi}$ come in complex conjugate pairs. The issue arises because, for homogeneous measurements, all the entries in the columns of $\boldsymbol{\Psi}$ come from the top, or the bottom partition, of the eigenvectors and are, therefore, from the

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latent vector. The latent vector is nearly real in the open loop (exactly real if the damping is assumed classical), and while this is not so in the closed loop, since the system is no longer self-adjoin, the coherence between the real and the imaginary parts is generally high. High coherence translates into poor conditioning; which implies a high norm of the inverse of $\boldsymbol{\Psi}$, hence resulting in large feedback gains and, as such, in large movements of the unplaced poles that, with near certainty, lead to instability. A solution that suggests itself is to use sensors such that the columns in $\boldsymbol{\Psi}$ contain entries from the displacement and velocity partitions of the eigenvectors, and numerical examination shows that this approach does mitigate instability. Analysis also reveals, however, that a mixed sensing scheme is not the ideal solution; not just because it is less desirable from a practical perspective, but because it does not offer a convenient means to affect the critical point, namely, the tradeoff between sensitivity and stability.

The pole placement problem by output feedback has long been known to be nonlinear in nature and it remains, in spite of significant progress, only partially solved [5–8]. In particular, it is known that for $n \leqslant m \cdot r$, where m, r, and n are the number of outputs, inputs and the system order, the system is pole assignable, although no effective algorithm to determine a gain that attains a given desired eigenstructure is available. For $n > m \cdot r$, some eigenstructures can be realized and others cannot, and it is not known how to distinguish between them; what is known, and for which there is an effective algorithm, is how to find a gain that places m poles with right-side eigenvector amplitudes generically fixed at r locations or r poles with left-side eigenvectors fixed at m locations [9,10]. Also available is a scheme that trades flexibility in the placement of r eigenvectors to allow placement of $m + r - 1$ poles [11]. Note that inasmuch as pole derivatives depend on the right- and left-side eigenvectors, design for sensitivity is an eigenstructure assignment problem, not just a pole placement one. Needless to say, the literature on the use of output feedback for stabilization, tracking, or regulation of linear systems is extensive and a survey can be found in [12]. References on its use in the control of nonlinear systems can be found in [13,14], among others.

The rest of the paper is organized as follows: following this introduction, the standard SVD eigenstructure design scheme and the computation of the closed-loop eigenvalue derivatives are reviewed. The next section clarifies the behavior that leads to ubiquitous instability for homogeneous sensing and puts forth a solution that retains the option of equal sensor types. A numerical example and a brief concluding section close the paper.

2. Pole and eigenvector placement using output feedback

Consider a linear, time-invariant system in discrete time described by

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}_d\boldsymbol{x}_k + \boldsymbol{B}_d\boldsymbol{u}_k + \boldsymbol{B}_f\boldsymbol{f}_{\boldsymbol{k}} \tag{1}$$

$$\boldsymbol{y}_k = \boldsymbol{C}\boldsymbol{x}_k \tag{2}$$

operating under the influence of static output feedback of the form

$$\boldsymbol{u}_k = -\boldsymbol{G}\boldsymbol{y}_k \tag{3}$$

Here, $\boldsymbol{x}_{\mathrm{k}} \in \Re^{n\times 1}$ is the state, $\boldsymbol{y}_{\mathrm{k}} \in \Re^{m\times 1}$ is the output, $\boldsymbol{u}_{\mathrm{k}} \in \Re^{r\times 1}$ are the control inputs, and $\boldsymbol{f}_{\mathrm{k}} \in \Re^{z\times 1}$ is the exogenous loading, which may be stochastic, if from ambient sources, or deterministic, if actuators are used to deliver it. $\boldsymbol{G} \in \Re^{r\times m}$ is the controller gain while $\boldsymbol{A}_{\mathrm{d}} \in \Re^{n\times n}$, $\boldsymbol{B}_{\mathrm{d}} \in \Re^{n\times r}$, $\boldsymbol{B}_{\mathrm{f}} \in \Re^{n\times z}$ and $\boldsymbol{C} \in \Re^{m\times n}$ are the system matrices, and we assume throughout that $\{\boldsymbol{A}_{\mathrm{d}}, \boldsymbol{B}_{\mathrm{d}}\}$ is controllable and $\{\boldsymbol{A}_{\mathrm{d}}, \boldsymbol{C}\}$ is observable. Eq. (2) holds directly when measurements are displacements, velocities, or non-collocated accelerations and can be used in the case of collocated accelerations if the direct transmission matrix is known and its contribution is subtracted from the measurements. Substituting Eq. (2) into Eq. (1) one finds that the closed-loop system is

$$\boldsymbol{x}_{k+1} = (\boldsymbol{A}_d - \boldsymbol{B}_d\boldsymbol{G}\boldsymbol{C})\boldsymbol{x}_k + \boldsymbol{B}_f\boldsymbol{f}_k \tag{4}$$

Let, $\boldsymbol{\Lambda} = \{\, \lambda_1 \quad \cdots \quad \lambda_p \,\}$ be the location of p closed-loop poles. For $\boldsymbol{G}$, to be real, $\boldsymbol{\Lambda}$ must be closed under conjugation, and it is evident that a necessary condition for stability is $\|\lambda_j\| \leqslant 1$. Since the controller is implemented in discrete time (DT), it is appropriate to extract the gain operating in DT so that the effect of the inter-sample behavior of the control can be considered [15]. The closed-loop eigenvalue problem writes

$$(\boldsymbol{A}_d - \boldsymbol{B}_d\boldsymbol{G}\boldsymbol{C})\boldsymbol{\psi}_j = \boldsymbol{\psi}_j\lambda_j \tag{5}$$

from where

$$[\,\boldsymbol{A}_d - \boldsymbol{I}\cdot\lambda_j \quad -\boldsymbol{B}_d\,]\begin{Bmatrix} \boldsymbol{\psi}_j \\ \boldsymbol{G}\boldsymbol{C}\psi_j \end{Bmatrix} = \boldsymbol{0} \tag{6}$$

Defining

$$\boldsymbol{V}_j = Null([\,\boldsymbol{A}_d - \boldsymbol{I}\cdot\lambda_j \quad -\boldsymbol{B}_d\,]) = \begin{bmatrix} \boldsymbol{S}_j \\ \boldsymbol{Q}_j \end{bmatrix} \tag{7}$$