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Technical communique

An implicit sequential algorithm for solving coupled Lyapunov equations of continuous-time Markovian jump systems[☆]

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ABSTRACT

In this paper, an implicit sequential algorithm is presented for solving coupled Lyapunov matrix equations of continuous-time Markovian jump linear systems. First, some existing iterative algorithms which can be utilized to solve the coupled Lyapunov matrix equations are reviewed and discussed. Next, based on the existing parallel iterative algorithm, an implicit sequential algorithm is proposed by using the latest updated information. The proposed algorithm fills the current gap of implicit algorithms for solving continuous coupled Lyapunov matrix equations. It is shown that the proposed algorithm with zero initial conditions can monotonically converge to the unique positive definite solutions of the coupled Lyapunov matrix equations if the associated Markovian jump system is stochastically stable. Moreover, a necessary and sufficient condition is established for the proposed algorithm to be convergent. The algorithm presented in this paper has much better convergence performance than other existing iterative algorithms and requires less storage capacity. Finally, a numerical example is given to show the effectiveness of the proposed algorithm.

1. Introduction

Markovian jump systems are a family of multi-modal systems and the mutual transitions between these modes are governed by a Markov chain. This kind of systems can be used to model some dynamic systems subject to abrupt changes in their structure and parameters due to, for instance, component failure or repairs, abrupt environment changes and changing subsystem interconnections (Ji & Chizeck, 1990). They have a wide range of applications, such as network control systems (Zhang, Shi, & Wang, 2013) and fault tolerant control systems (Li, Gao, Shi, & Zhao, 2014). Due to this, considerable attention has been attracted from many researchers. For example, the filter design problem of Markovian jump systems with time delays was investigated in Xu, Chen, and Lam (2003) and the stabilization problem of Markovian jump systems was considered in Xiong and Lam (2006). In Ji and Chizeck (1990), stochastic controllability and stabilizability were studied for continuous-time Markovian jump linear systems. In Mariton (1988), the moment stability was studied for this kind of systems. Stochastic stability of jump linear systems and the relationship among various moment and sample path stability were studied in Feng, Loparo, Ji, and Chizeck (1992). It was shown in Mariton (1988) and Feng et al. (1992) that both the stochastic and moment stability can be characterized by the existence of unique positive definite solutions of the coupled Lyapunov matrix equations (CLMEs). In addition, a CLMEs based criterion was given in Ji and Chizeck (1990) for the stochastic controllability.

The aforementioned facts imply that the CLMEs play an important role in stability analysis and stabilizing controller design. Therefore, much effort has been made to develop some approaches for solving them. A direct and natural way is to use the Kronecker product to transform them into matrix–vector linear equations (Jodar & Mariton, 1987). Obviously, this approach suffers from high dimensions. Another efficient way to solve coupled matrix equations is through iteration. In Borno (1995), a parallel iterative algorithm was proposed for solving the continuous CLMEs. It was pointed out in Borno (1995) that the sequences generated by the algorithm are monotonically increasing, and upper bounded. In this algorithm, some independent standard Lyapunov matrix equations need to be solved at each iteration step. The parallel iterative algo-

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rithm was then extended in Borno and Gajic (1995) to solve the discrete CLMEs with the requirement of zero initial conditions. It was proven in Wang, Lam, Wei, and Chen (2008) that the iteration sequences generated by the algorithm in Borno and Gajic (1995) are also convergent without this requirement. In Wang et al. (2008), an iterative algorithm was constructed for solving the discrete CLMEs. In this algorithm, the iteration sequence converges if the spectral radius of an augmented matrix is less than one. From an optimization point of view, a gradient based iterative algorithm was developed in Zhou, Lam, and Duan (2008) for solving the discrete CLMEs. In Zhou et al. (2008), a necessary and sufficient condition guaranteeing the convergence of the algorithm was established, and the optimal step size maximizing the convergence rate of the algorithm was given explicitly. This gradient based iterative algorithm was then extended in Zhou, Duan, and Li (2009) to solve coupled matrix equations, including the continuous CLMEs as a special case. Based on the positive operator theory, two iterative algorithms were established in Li, Zhou, Lam, and Wang (2011) for both discrete and continuous Lyapunov equations associated with Itô stochastic systems with Markovian jumps, which include the Markovian jump systems as a special case. Recently, by taking advantage of the algorithms in Borno and Gajic (1995) and Wang et al. (2008) and using the updated variables in the current step for estimation of other variables, two types of iterative algorithms were proposed in Wu and Duan (2015) for solving the discrete CLMEs. It was shown that the algorithms in Wu and Duan (2015) converge much faster than those in Borno and Gajic (1995) and Wang et al. (2008).

On solving the discrete CLMEs, the parallel iterative algorithm in Borno and Gajic (1995) is explicit. In Wang et al. (2008), an implicit iterative algorithm motivated by the solution of single discrete Lyapunov equation was presented. Based on the algorithms in Borno and Gajic (1995) and Wang et al. (2008), some explicit and implicit iterative algorithms were developed by using the *latest updated information* in Wu and Duan (2015). On the other hand, for solving the continuous CLMEs, the parallel iterative algorithm in Borno (1995) is implicit without using the *latest updated information*. Therefore, in this brief communique, we will develop an implicit iterative algorithm for solving the continuous CLMEs by using the *latest updated information* proposed in Wu and Duan (2015) and fill the current gap for continuous-time Markovian jump systems.

For the iterative algorithm in Borno (1995), the iteration sequence is updated by only using the information in the last step. However, in the current step some variables have been updated before the estimates of the other variables are calculated. In fact, these updated information for some variables can be utilized effectively for other variables to be updated. Based on this idea, we will investigate the iterative algorithm for solving the continuous CLMEs by revising the iterative algorithm in Borno (1995). This paper can be viewed as an implicit investigation for continuous-time systems corresponding to the results of the implicit algorithm in Wu and Duan (2015) for discrete-time systems. We should address here that the CLMEs for continuous-time and discrete-time systems are quite different in general and the techniques used in proofs for many dual version results are also different due to the representation difference. The essential idea of the *latest updated information* in this paper is based on Wu and Duan (2015).

Throughout this paper, for a matrix $A \in \mathbb{R}^{n\times n}$, A^{T}, $\rho(A)$, $\sigma_{\max}(A)$ and $\sigma_{\min}(A)$ denote its transpose, spectral radius, maximal singular value and minimal singular value, respectively; $A > 0$ ($A < 0$, respectively) represents that A is positive definite (negative definite, respectively). For two matrices M and N with the same dimension, $M < N$ means $M - N < 0$. $\|\cdot\|$ denotes the Frobenius norm. $\otimes$ represents the Kronecker product of two matrices. E represents the mathematical expectation. I_n stands for the identity matrix of size $n \times n$. The vectorization operator vec is defined as $\mathrm{vec}(A) = \begin{bmatrix} a_1^{\mathrm{T}} & a_2^{\mathrm{T}} & a_3^{\mathrm{T}} & \cdots & a_n^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$ where $A = \begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix}$.

2. Previous results

Consider the following continuous-time Markovian jump linear system

$$\dot{x}(t) = A_{r(t)}x(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state of the system, $A_{r(t)} \in \mathbb{R}^{n\times n}$ is the mode-dependent system matrix, and $r(t)$ is a Markov random process that takes values in a finite discrete set $\mathcal{N} = \{1, 2, \ldots, N\}$. The dynamics of the probability distribution of the Markov chain are determined by the differential equation $\dot{\pi}(t) = \pi(t)P$, where π is an N-dimensional row vector of unconditional probabilities, and P is the transition rate matrix given by $\left[p_{ij}\right]_{N\times N}$. The matrix P has the properties that $p_{ii} < 0$, $p_{ij} \geq 0$, for $i \neq j$, and $\sum_{j=1}^{N} p_{ij} = 0$, for $i \in \mathcal{N}$. For the system (1), let the initial condition be $x(0) = x_0$ and $r(0) = r_0$, then the definition of stochastic stability can be given as follows.

Definition 1 (*Feng et al., 1992*)**.** The Markovian jump system (1) is said to be stochastically stable if for any initial condition $x_0 \in \mathbb{R}^n$ and $r_0 \in \mathcal{N}$, there holds

$$\mathrm{E}\left\{\int_0^{\infty} \|x(t)\|^2 \mid x_0, r_0\right\} < \infty. \tag{2}$$

The coupled Lyapunov matrix equations (CLMEs) corresponding to the system (1) are

$$A_i^{\mathrm{T}}K_i + K_iA_i + Q_i + \sum_{j=1}^{N} p_{ij}K_j = 0, \quad Q_i > 0, i \in \mathcal{N}, \tag{3}$$

where i indicates that the system is in the i-th mode, namely, $A_{r(t)} = A_i$. It has been shown in Feng et al. (1992) that the Markovian jump system (1) is stochastically stable if and only if the CLMEs (3) have positive definite solutions K_i, $i \in \mathcal{N}$, for any positive definite matrices Q_i, $i \in \mathcal{N}$. On the other hand, the CLMEs associated with the discrete-time Markovian jump system can be given by

$$A_i^{\mathrm{T}}\left(\sum_{j=1}^{N} p_{ij}K_j\right)A_i - K_i + Q_i = 0, \quad Q_i > 0, i \in \mathcal{N}. \tag{4}$$

Remark 1. Due to the representation difference, the techniques used to establish the iterative solutions of (3) and (4) are quite different. For (4), as the matrices K_i, $i \in \mathcal{N}$ to be determined appear alone, it is easy to give the explicit iteration directly (see Wu and Duan (2015) for details). In contrast, regarding (3), K_i, $i \in \mathcal{N}$ appear with a matrix in their left or right sides. As a result, it is not an easy task to present their explicit iterative solutions. In addition, there are few elegant results reported in the literature on the explicit solutions of (3). By using the Kronecker product, the explicit iterative solutions of (3) were obtained in Jodar and Mariton (1987) and Zhang and Ding (2014). However, such solutions suffer from computational difficulties due to high dimensionality of the associated matrices. Another method to get the explicit iterative solutions of (3) was provided in Li et al. (2011) by using an auxiliary transformation. In Li et al. (2011), the continuous stochastic Lyapunov equations were transformed into some equivalent discrete Lyapunov equations. A drawback of this approach is that it requires additional computation. One can observe that the investigation of the CLMEs for continuous-time systems is far behind from that for discrete-time systems. To the best knowledge of the authors, the iterative solutions of the continuous CLMEs have not been well investigated.