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A highly efficient compressed sensing algorithm for acoustic imaging in low signal-to-noise ratio environments

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ABSTRACT

We study the acoustic imaging in low signal-to-noise ratio (SNR) environments with compressed sensing (CS) and microphone arrays. In this work, we propose an OMP-SVD method which combines the orthogonal matching pursuit (OMP) method of CS and the singular value decomposition (SVD). The performance of the proposed OMP-SVD method is compared with the CBF method, the OMP method and the l_1-SVD method. In terms of the CPU time, the proposed method is highly efficient like the CBF method and the OMP method, and much more efficient than the l_1-SVD method. In terms of the accuracy of the source maps, the OMP-SVD method can locate the sources exactly for the SNR as low as −10 dB and the frequency as low as 2000 Hz, while the other three different methods can only locate the sources when the SNR is greater than or equal to 5 dB. In addition, we find that the proposed method can obtain good performance when the target sparsity K_T is overestimated and there is basis mismatch. Finally, a gas leakage experiment was conducted to verify the performance of the OMP-SVD method in practical application. The results show that the OMP-SVD method is robust in low SNR environments.

1. Introduction

Acoustic imaging has been an indispensable technique for locating acoustic sources in realistic environments for many mechanical engineering applications with the strong background or environment noise [1,2]. Microphone arrays combined with beamformers are by far the most prevalent technique for acoustic imaging due to its robustness in noise situations and its simplicity. Conventional beamformer (CBF) derived from plane-wave model [3] suffers from poor spatial resolution and pronounced side lobes contamination [4–6]. In order to improve the spatial resolution and suppress the effect of side lobes, many deconvolution methods have been proposed. The CLEAN algorithm [7] has been proposed to extract the strong sources from the blurry CBF maps, which can obtain high-resolution maps. However, the CLEAN algorithm may leave out the weak sources when the background noise is strong. Brooks and Humphreys [4] proposed a complete Deconvolution Approach for the Mapping of Acoustic Sources (DAMAS) algorithm. The DAMAS algorithm not only can improve the spatial resolution, but also can reduce or even eliminate the side lobes efficiently. But the primary weaknesses of the DAMAS algorithm lie in its computational expensiveness compared with the CBF method and its high sensitivity to background noise [5]. The MUltipe SIgnal Classification (MUSIC) method has been proposed by Schmidt [8] to improve the spatial resolution of the CBF method.

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However, the MUSIC method requires the high signal-to-noise ratio (SNR) environment and knowing the exact number of sources to make the subspace separation [5].

Compressed sensing (CS) is a recently developed revolutionary theory in signal processing [9–13]. CS shows that the signal can be reconstructed with convex optimization algorithms or non-convex optimization algorithms from very few measurements when the signal is compressible or sparse [14]. CS has also been employed in a wide range of applications including acoustic imaging [6,15–18]. Comerford et al. [19] applied CS theory in the presence of missing data and it has been shown to be highly effective in reconstructing the power spectrum in both stationary and non-stationary processes. Yang and Nagarajaiah [20] presented a new output-only modal identification method based on a combination of CS and BSS (CP) using the non-uniform low-rate random sensing framework. Convex optimization algorithms have been applied to obtain acoustic imaging. Bai et al. [21,22] examined the application of CS + convex optimization (CVX) method in acoustical array signal processing, and formulated the acoustic sources separation problem. Zhong et al. [16] proposed a CS beamforming method, which is based on sampling covariance matrix for the two-dimensional acoustic imaging. The Basis Pursuit (BP) algorithm, a convex optimization algorithm, has been applied in the context of CS by Simard and Antoni [6], to solve two-dimensional acoustic source identification from a limited number of measurements by a microphone array. In the above studies, the researchers have found that the main characteristic of CS method is super-resolution compared with beamformers. However, convex optimization algorithms need relatively high computational effort, and are subject to the Restricted Isometry Property (RIP). In order to improve the computational efficiency of CS method, we have applied Orthogonal Matching Pursuit (OMP) algorithm, a kind of greedy algorithm, to acoustic imaging [17,18]. At the same time, we found that results of the method are unstable in low SNR environments. SVD, the singular value decomposition, has been widely used in the field of signal processing combined with sparse recovery algorithms. Park et al. [23] proved that the SVD of a compressed data matrix can return accurate estimates of the true mode shape vectors through simulation with real-valued dataset. Yang et al. [24] applied SVD analysis of the noisy strain matrix and discarded (set to zero) those small singular values that are contaminated by noise to get clearer strain maps. Malioutov et al. [25] developed an approach, l_1-SVD, based on the singular value decomposition (SVD) to combine multiple samples and the use of second-order cone programming for source localization. The method is robust in low SNR environments. However, it is relatively computationally expensive and difficult to choose the regularization parameter, which is similar to the drawbacks of BP algorithm.

In order to reduce the computational cost and increase the robustness in low SNR acoustic environments, we propose an OMP-SVD method in this work, which combines the OMP method and SVD. We compare the benefits and limitations of several methods for low SNR acoustic imaging. We examine the applicability of the OMP-SVD method under different SNR and different frequencies. The performance of our proposed method is also investigated when the target sparsity K_T [14] is overestimated and there is basis mismatch. In addition, we have also performed physical experiments to verify the feasibility of OMP-SVD method in acoustic imaging in low SNR environments.

This paper is organized as follows: Section 2 describes the observation model of acoustic signal propagation. Then our proposed method is presented in Section 3. Subsequently, the performance of the method is compared with others by simulations in Section 4. More, the analysis of the OMP-SVD method is also shown in this section. Section 5 provides a gas leakage experiment and compares the real experimental data with the simulation results from the proposed method. Finally, we conclude this paper in Section 6.

2. Observation model

Fig. 1 illustrates the model of acoustic signal propagating from the source plane which is h away from the planar microphone array with M sensors, which are located at known positions $\overline{\mathbf{P}} = [\overline{\mathbf{p}}_1, \ldots, \overline{\mathbf{p}}_M]^{\mathrm{T}}$, where $[\bullet]^{\mathrm{T}}$ denotes the transpose operator. The source plane is discretized into $N = u \times u$ equidistant grids at known discrete positions $\mathbf{P} = [\mathbf{p}_1, \ldots, \mathbf{p}_N]$. The measurements of sound pressure at M microphones of the array in the frequency domain are $\mathbf{Y}(f) = [y_1, \ldots, y_M]^{\mathrm{T}}$. The unknown vector $\mathbf{X}(f) = [x_1, \ldots, x_N]^{\mathrm{T}}$ comprises the source strengths at all N grid nodes. The nth element of the vector $\mathbf{X}$ equals to zero if there is no source at the nth grid node. The pressure field at the mth microphone is given by:

$$y_m = \sum_{n=1}^{N} \frac{x_n \cdot e^{-jkr_{mn}}}{4\pi r_{mn}}, \tag{1}$$

where $r_{mn} = \|\overline{\mathbf{p}}_m - \mathbf{p}_n\|$ denotes the distance between the mth microphone and the nth grid node, $k = \omega/c$ is the wave number with c being the sound speed, j is the imaginary unit, $\omega = 2\pi f$ is the angular frequency with f being the desired frequency, and x_n is the source strength of the nth grid node.

The model can be compactly expressed in matrix form:

$$\mathbf{Y} = \mathbf{A}\mathbf{X},$$

$$\mathbf{Y} = \begin{bmatrix} y_1 \\ \vdots \\ y_M \end{bmatrix}, \quad \mathbf{A} = \frac{1}{4\pi} \begin{bmatrix} \frac{e^{-jkr_{11}}}{r_{11}} & \cdots & \frac{e^{-jkr_{1N}}}{r_{1N}} \\ \vdots & \ddots & \vdots \\ \frac{e^{-jkr_{M1}}}{r_{M1}} & \cdots & \frac{e^{-jkr_{MN}}}{r_{MN}} \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}, \tag{2}$$