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On evolutionary system identification with applications to nonlinear benchmarks

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ABSTRACT

This paper presents a record of the participation of the authors in a workshop on nonlinear system identification held in 2016. It provides a summary of a keynote lecture by one of the authors and also gives an account of how the authors developed identification strategies and methods for a number of benchmark nonlinear systems presented as challenges, before and during the workshop. It is argued here that more general frameworks are now emerging for nonlinear system identification, which are capable of addressing substantial ranges of problems. One of these frameworks is based on evolutionary optimisation (EO); it is a framework developed by the authors in previous papers and extended here. As one might expect from the 'no-free-lunch' theorem for optimisation, the methodology is not particularly sensitive to the particular (EO) algorithm used, and a number of different variants are presented in this paper, some used for the first time in system identification problems, which show equal capability. In fact, the EO approach advocated in this paper succeeded in finding the best solutions to two of the three benchmark problems which motivated the workshop. The paper provides considerable discussion on the approaches used and makes a number of suggestions regarding best practice; one of the major new opportunities identified here concerns the application of grey-box models which combine the insight of any prior physical-law based models (white box) with the power of machine learners with universal approximation properties (black box).

1. Introduction

In March of 2016, an interesting meeting on the subject of nonlinear system identification (NLSI) took place at Vrije Universiteit Brussel (VUB) in the Belgian capital. The meeting was interesting for two reasons; in the first case, it was organised with the intention of bringing together experts from the disciplines of electrical engineering, mechanical engineering and machine learning, in order to draw out common elements of best practice for nonlinear system modelling/identification, and also to exploit any potential synergies. The second feature of interest was that the meeting was organised around the discussion of three benchmarks for NLSI, each designed in such a way as to challenge theory and practice in specific ways.

The three benchmark problems were as follows:

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- **A Bouc-Wen Hysteretic System**. The NLSI challenges of this benchmark were associated with the fact that the system of interest had an unmeasurable state in its equations of motion, and the fact that the model form was not linear in the parameters. The system equations were encoded in a Matlab p-file, which allowed participants complete freedom in choosing the form of the excitation used for identification. The data were thus generated by computer simulation, although noise was added to the response in the p-file to give an element of realism.
- **A Wiener-Hammerstein System with Process Noise**. The main challenge associated with this benchmark was that the system was a block-structured system where significant noise was added to an internal state. The system was encoded in an electronic circuit and was thus experimental (at least from an electrical engineering point of view). Although participants did not have the freedom to completely experiment with excitation signals, they were allowed to propose signals, which were then used in a number of measurement campaigns in order to collect data for the benchmark exercise.
- **A Cascaded Tanks System**. This was an experimental liquid level system, in which fluid passed between two tanks. The main challenges were that an unmeasured state was present again, but mainly that the record of data for NLSI was very short. Further challenges arose due to the overflow of the tanks, which introduced a hard saturation nonlinearity and some uncertainty in the form of process noise. Participants were not given any control of the experiment in this particular case.

This paper is a record of the participation of a team of University of Sheffield (UoS) academics and researchers. It comprises a summary of a keynote presentation by one of the authors, followed by detailed descriptions of how the team attempted to solve the benchmark problems. It is argued here that general frameworks are beginning to emerge for NLSI, which are capable of addressing ranges of disparate problems. Two of the main candidates for such a general framework are those based on evolutionary optimisation (EO) and Bayesian inference. In fact, the algorithms applied here were taken from the EO approach developed by the authors over a number of years and extended in order to address the benchmarks. The power of the EO framework is clearly evidenced by the fact that it provided the best solutions to two out of the three benchmark problems at the focus of the workshop. As one might expect from the 'no-free-lunch' theorem for optimisation [1], it would be surprising if a single variant of the EO algorithm stood out as the overall best choice, so the viewpoint here has been to present a number of possibilities (reflecting the slightly different tastes of the authors and illustrating the range). Although the EO approach is favoured in this paper, the Bayesian framework for NLSI is also very powerful and is being pursued by the authors; however, there is simply not room here to compare the two frameworks. If the reader is interested in seeing how modern Bayesian methods can contribute to NLSI they can consult the references in the following section.

One of the main contributions of this paper is to highlight and develop the idea of using *grey-box* models for NLSI. Grey box models combine the insight of a physics-based (white box) model with the explanatory power of machine learners (black box) which have universal approximation/representation properties. In fact, the grey-box model presented here for Benchmark Three also combines the power of the EO and Bayesian approaches by using a Gaussian process NARX (Nonlinear AutoRegressive with eXogeneous inputs) model to capture behaviour missed by the physical model and to thus substantially improve predictions.

It is important to note two facts. The first is that the paper has been formed in order to give an honest account of the identification results, as presented at the workshop; it deliberately does not contain any results which exploit lessons learned *during or after* the meeting, although those lessons have led to a great deal of progress for the participants since. The second fact to note, is that four separate studies are presented here, each carried out by separate subgroups of the UoS team; this means that the studies may reflect slightly different views on the *practice* of NLSI – the authors are all in general agreement about the aims, objectives and importance of the subject. The amount of ground covered here also means that the paper is rather lengthy; this is sadly unavoidable if it is to reflect properly the weight of the work conducted.

The layout of the paper is as follows: the following section summarises one of the workshop keynotes, and discusses the question of whether NLSI can be reduced to a problem in machine learning. The three subsequent sections, outline in turn, how the authors approached the three VUB benchmark problems.

2. Is system identification simply machine learning?

2.1. Introduction

The material of this section was originally the subject of a keynote at the VUB Workshop; as a result, its remit is broader than the material which follows, which presents studies of the specific benchmark exercises. However, the discussion will touch on various issues which surface during the detailed studies and will also attempt to capture aspects of current thinking in terms of NLSI within the Engineering Dynamics community. In order to faithfully cover what was discussed in the keynote, it will be necessary to go over a little previously-published ground; however, this will also help to make the paper more self-contained.

Historically, one could argue that the main developments in the general theory of linear SI have come from the electrical engineering and control communities. This work resulted in a comprehensive and rigorous body of material which is visible through classic texts and monographs like [2,3]. Although general ideas from SI were certainly adopted by the Engineering Dynamics community, the main developments there are associated with a specific method – *modal analysis* [4]. Modal analysis arose naturally as a result of the fact that linear engineering dynamics is concerned with a specific system of second-order differential equations, derived from Newton's second law, and usually expressed in the matrix form,