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Stochastic sensor scheduling via distributed convex optimization☆

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ABSTRACT

In this paper, we propose a stochastic scheduling strategy for estimating the states of N discrete-time linear time invariant (DTLTI) dynamic systems, where only one system can be observed by the sensor at each time instant due to practical resource constraints. The idea of our stochastic strategy is that a system is randomly selected for observation at each time instant according to a pre-assigned probability distribution. We aim to find the optimal pre-assigned probability in order to minimize the maximal estimate error covariance among dynamic systems. We first show that under mild conditions, the stochastic scheduling problem gives an upper bound on the performance of the optimal sensor selection problem, notoriously difficult to solve. We next relax the stochastic scheduling problem into a tractable suboptimal quasi-convex form. We then show that the new problem can be decomposed into coupled small convex optimization problems, and it can be solved in a distributed fashion. Finally, for scheduling implementation, we propose centralized and distributed deterministic scheduling strategies based on the optimal stochastic solution and provide simulation examples.

1. Introduction

In this paper, we consider the problem of scheduling the observations of independent targets in order to minimize the tracking error covariance, but when only one target can be observed at a given time. This problem captures many interesting tracking/estimation application problems. As a motivational example, consider N independent dynamic targets, spatially distributed in an area, that need to be tracked (estimated) by a single (mobile) camera sensor. The camera has limited sensing range and therefore it needs to zoom in on, or be in proximity of, one of the targets for obtaining measurements. Under the assumption that the switching time among the targets is negligible, then we need to find a visiting sequence in order to minimize the estimate error.

Another case is when a set of N mobile surveillance devices need to track N geographically-separated targets, where each target is tracked by one assigned surveillance device. However, the sensing/measuring channel can only be used by one estimator at the time (e.g. sonar range-finding Cremean et al., 2002). Then, we need to design a scheduling sequence of surveillance devices for accurate tracking.

1.1. Related work and contributions of this paper

There has been considerable research effort devoted to the study of sensor selection problems, including sensor scheduling (Alriksson & Rantzer, 2005; Bitar, Baeyens, & Poolla, 2009; Gupta, Chung, Hassibi, & Murray, 2006; Joshi & Boyd, 2009; Li & Elia, 2011; Liang, Tang, & Zhu, 2007; Lin & Wang, 2013; Ny, Feron, & Dahleh, 2011; Shi, Johansson, & Murray, 2007; Srivastava, Plarre, & Bullo, 2011; Tiwari, Jun, Jeffcoat, & Murray, 2005; Vasanthi & Annadurai, 2006) and sensor coverage (Acar & Choset, 2002; Choset, 2001; Cortes, 2010; Cortes, Martinez, Karatas, & Bullo, 2004; Gupta et al., 2006; Hussein & Stipanovi, 2007). This trend has been inspired by the significance and wide applications of sensor networks. As the literature is vast, we list a few results which are relevant to this paper. The sensor scheduling problem mainly arises from minimization of two relevant costs: sensor network energy consumption and estimate error. On the one hand, (Liang et al., 2007; Vasanthi & Annadurai, 2006) and (Bitar et al., 2009), see also reference therein, have proposed various efficient sensor scheduling algorithms to minimize the sensor network energy consumption and consequently maximize the network lifetime. On the other hand, researchers have proposed many tree-search based

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sensor scheduling algorithms (mostly in conjunction with Kalman filtering) to minimize the estimate error (Alriksson & Rantzer, 2005; Lin & Wang, 2013; Tiwari et al., 2005), e.g. sliding-window, thresholding, relaxed dynamic programming, etc. By taking both sensor network lifetime and estimate accuracy into account, several sensor tree-search based scheduling algorithms have been proposed in Shi et al. (2007), Shi, Capponi, Johansson, and Murray (2008). In Joshi and Boyd (2009), the authors have formulated the general sensor selection problem and solved it by relaxing it to a convex optimization problem. The general formulation therein can handle various performance criteria and topology constraints. However, the framework in Joshi and Boyd (2009) is only suitable for static systems instead of dynamic systems which are mostly considered in the literature.

In general, deterministic optimal sensor selection problems are notoriously difficult. In this paper, we propose a stochastic scheduling strategy. At each time instant, a target is randomly chosen to be measured according to a pre-assigned probability distribution. We find the optimal pre-assigned distribution that minimize an upper bound on the expected estimate error covariance (in the limit) in order to keep the actual estimate error covariance small. Compared with algorithms in the literature, this strategy has low computational complexity, it is simple to implement and provides performance guarantee on the general deterministic scheduling problem. Of course, the reduction of computational complexity comes at the expenses of degradation of the ideal performance. However, in many situations the extra computational complexity cost may not be justified. Further this strategy can easily incorporate extra constraints on the scheduling design, which might be difficult to handle in existing algorithms (e.g. tree-search based algorithms).

Our work is related to Gupta et al. (2006), Mo, Garone, Casavola, and Sinopoli (2011) and Ny et al. (2011). Gupta et al. (2006) introduces stochastic scheduling to deal with sensor selection and coverage problems, and Mo et al. (2011) extends the setting and results in Gupta et al. (2006) to a tree topology. Although we also adopt the stochastic scheduling approach, the problem formulation and proposed algorithms of this paper are different from Gupta et al. (2006), Mo et al. (2011). In particular, we consider different cost functions and design distributed algorithms that provide optimal probability distributions. Ny et al. (2011) has considered a scheduling problem in continuous-time and proposed a tractable relaxation, which provides a lower bound on the achievable performance, and an open-loop periodic switching strategy to achieve the bound in the limit of arbitrarily fast switching. However, besides the difference in the formulations, their approach does not appear to be directly extendable to the discrete-time setting. In summary, our main contributions include:

(1) We obtain a stochastic scheduling strategy with performance guarantee on the general deterministic scheduling problem by solving distributed optimization problems.
(2) For scheduling implementation, we propose both centralized and distributed deterministic scheduling strategies.

1.2. Notations and organization

Throughout the paper, A' is the transpose of matrix A. $Ones(n, n)$ implies an $n \times n$ matrix with 1 as all its entries. $Diag(V)$ denotes a diagonal matrix with vector V as its diagonal entries. $M \succeq 0$ (or $M \in S_+$) and $M \succ 0$ (or $M \in S_{++}$) respectively implies matrix M is positive semi-definite and positive definite where S_+ and S_{++} represent the positive semi-definite and positive definite cones. For a matrix A, if the block entry $A_{ij} = A'_{ji}$, we use $(\cdot)$ in the matrix to present block A_{ji}. The trace of a square matrix is denoted by $Tr(\cdot)$.

The paper is organized as follows. In Section 2, we mathematically formulate the stochastic scheduling problem. In Section 3, we develop an approach and a distributed computing algorithm to solve the optimization problem. In Section 4, we present some further results and the extensions of our model. In Section 5, we consider the scheduling implementation problem. At last, we present simulations to support our results.

2. Sensor scheduling problem setup

Consider a set of N DTLTI systems (targets) evolving according to the equations

$$x_i[k+1] = A_i x_i[k] + w_i[k] \quad i = 1, 2, \ldots, N \tag{1}$$

where $x_i[k] \in \mathbb{R}^{n_i}$ is the process state vector and $w_i[k] \in \mathbb{R}^{n_i}$ is assumed to be an independent Gaussian noise with zero mean and covariance matrix $Q_i \succ 0$. The initial state $x_i[0]$ is assumed to be an independent Gaussian random variable with zero mean and covariance matrix $\pi_i[0]$. In practice, each DTLTI system modeled above may represent the dynamic change of a local environment, the trajectory of a mobile vehicle, the varying states of a manufactory machine, etc. As a result of the sensor's limited range of sensing or the congestion of the sensing channel, at time instant k, only one system can be observed as

$$\tilde{y}_i[k] = \xi_i[k](C_i x_i[k] + v_i[k]) \tag{2}$$

where $\xi_i[k]$ is the indicator function indicating whether or not the system i is observed at time instant k, and accordingly we have constraint[2] $\sum_{i=1}^{N} \xi_i[k] = 1$. $v_i[k] \in \mathbb{R}^{p_i}$ is the measurement noise, which is assumed to be independent Gaussian with zero mean and covariance matrix $R_i \succ 0$.

Assumption 1. For all $i \in \{1, 2, \ldots, N\}$, the pair $(A_i, Q_i^{1/2})$ is controllable and the pair (A_i, C_i) is detectable.

Denote $\hat{x}_i[k]$ as the estimate at time k, obtained by a causal estimator for system i, which depends on the past and current observations $\{\tilde{y}_i[j]\}_{j=1}^{k}$. We begin by considering problem of minimizing (in the limit) the maximal estimate error. The problem can be formulated mathematically as

$$\begin{aligned} \min_{\hat{x}_i, \{\xi_i[j]\}_{j=1}^{\infty}} \max_i & \left(\limsup_{T\to\infty} \frac{1}{T} \sum_{k=1}^{T} \mathbb{E}[(x_i[k] - \hat{x}_i[k])'(x_i[k] - \hat{x}_i[k])] \right) \\ \text{s.t.} \quad & Equation: (1), (2), \quad i = 1, \ldots, N, \\ & \sum_{i=1}^{N} \xi_i[k] = 1. \end{aligned} \tag{3}$$

As the DTLTI systems are assumed to be evolving independently, then for a fixed $\{\xi_i[j]\}_{j=1}^{\infty}$ the optimal estimator for minimizing the estimate error covariance of system i $(i = 1, 2, \ldots, N)$ is given by a Kalman filter[3] whose process of prediction and update are presented as follows (Sinopoli et al., 2004). Firstly we define

$$\hat{x}_i[k|k] \triangleq \mathbb{E}[x_i[k] | \{\tilde{y}_i[j]\}_{j=1}^{k}]$$

$$P_i[k|k] \triangleq \mathbb{E}[(x_i[k] - \hat{x}_i[k|k])(x_i[k] - \hat{x}_i[k|k])' | \{\tilde{y}_i[j]\}_{j=1}^{k}]$$

$$\hat{x}_i[k+1|k] \triangleq \mathbb{E}[x_i[k+1] | \{\tilde{y}_i[j]\}_{j=1}^{k}]$$

$$P_i[k+1|k] \triangleq \mathbb{E}[(x_i[k+1] - \hat{x}_i[k+1|k])(x_i[k+1] - \hat{x}_i[k+1|k])' | \{\tilde{y}_i[j]\}_{j=1}^{k}].$$

[2] If we assume *at most* one target is chosen to be measured at each time instant, then we have $\sum_{i=1}^{N} \xi_i \le 1$. Without loss of generality, in this paper we consider the case that one out of N targets must be chosen at each time instant.

[3] This indicates that N parallel estimators, i.e., Kalman filters, are used for estimating N independent DTLTI systems.