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Evanescent Bloch waves and complex band structure in magnetoelectroelastic phononic crystals

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ABSTRACT

We investigate the complex band structure of elastic waves in a magnetoelectroelastic phononic crystal, consisting of a polymeric matrix reinforced by $BaTiO_3$-$CoFe_2O_4$ inclusions in a square lattice. We also study the influence of inclusion geometry - circular, square and rotated square on the band structure. The elastic wave propagation inside periodic structures consists of both propagating and evanescent modes. Accordingly, improved plane wave expansion ($\omega(\mathbf{k})$) and extended plane wave expansion ($\mathbf{k}(\omega)$) methods are formulated to solve the governing equations of motion of a magnetoelectroelastic solid considering wave propagation in the *xy* plane in order to obtain the propagating and evanescent modes. Complete band gaps between the XY and Z modes are observed for all types of inclusion. Piezoelectricity and piezomagnetism influence significantly the band gaps and the attenuation. There are many modes of the magnetoelectroelastic phononic crystal that are not identified by the $\omega(\mathbf{k})$ method. However, they are well identified by the $\mathbf{k}(\omega)$ method proposed. These complex modes are related to the evanescent nature of Bloch waves. The new findings of this study should be useful for vibration control using smart periodic structures with piezoelectricity and piezomagnetism.

1. Introduction

Recently, artificial periodic composites known as phononic crystals (PCs) have been quite studied [1–9]. They are normally composed by a periodic arrangement of two or more materials with different properties. The mismatch between the constituent materials may arise either from difference of material properties and/or geometry (density, elastic modulus, cross-sectional area) - continuum-scale theory, or of interatomic force constants and masses - atomic-scale theory.

PCs have received renewed attention because they exhibit phononic band gaps. There are no mechanical (elastic or acoustic) propagating waves in phononic band gaps, only evanescent waves. Laude and co-workers [10] mentioned that the occurrence of a band gap is not indicated by an absence of bands but by the evanescent character of Bloch waves. Instead of the traditional definition of a complete band gap, that is to say a range of frequency where there are no propagating modes exist, Laude *et al.* [10] proposed a definition whereby all Bloch waves must be evanescent within a complete band gap.

The novel physical properties of PCs arise from the possibility of creating phononic band gaps and negative refraction (phonon branches with negative group velocity) [11]. The ability of creating phononic band gaps is similar to the creation of electronic and photonic band gaps in semiconductors/insulators and photonic crystals [12,13], respectively. The physical

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origin of phononic and photonic band gaps can be understood at micro-scale from the classical wave theory to describe the Bragg and Mie resonances based on the scattering of mechanical and electromagnetic waves propagating within the crystal [14].

PCs have many applications, such as vibration isolation technology [7–9,15–19], acoustic barriers/filters [20–22], waveguides [23–26], noise suppression devices [27,28], surface acoustic devices [29], architectural design [30], sound shields [31], acoustic diodes [32], elastic/acoustic metamaterials [33,34,22,28,35–37], also known as locally resonant phononic crystals, and thermal metamaterials [38,39], also known as phononic thermocrystals or locally resonant phononic thermocrystals.

There are also smart PCs that have been studied, for instance, piezoelectric [40–56] and piezomagnetic [57–60] PCs. However, few studies focused on magnetoelectroelastic phononic crystals (MPCs) [61–66]. Wang et al. [61–64] established MPC models and tuning approaches on band gaps are given systematically. They illustrated the superior characteristics of mechanical-electric-magnetic coupling in order to produce effective tunable methods of elastic wave propagation. Their findings also show that MPCs have rich potential in the active control of elastic waves which can overcome the traditional ways with changing material filling ratios. Lan and Wei [65] studied the band gaps of a laminated MPC with graded interlayer, considering different gradient profiles between piezoelectric and piezomagnetic materials. They observed that the low-frequency band gaps are more sensitive to the propagation direction of the SH wave, while the high-frequency band gaps are more sensitive to the graded profiles of the interlayer. Guo et al. [66] investigated the effects of functionally graded interlayers on band structures of elastic waves in a 1D MPC. They showed that functionally graded interlayers have evident influences on the dispersion curves and band gaps.

Furthermore, complex band structures are commonly discussed in the study of wave propagation in PCs [10,67–78], since they have the information of spatial wave propagation as well as spatial attenuation of elastic waves. The analysis of complex band structures may also contribute to a better understanding of band gap formation since tracing the evanescent Bloch waves becomes possible [73]. Wang et al. [72] presented a study of acoustic wave propagation in 1D locally resonant PCs made of acoustic resonators grafted onto a waveguide. They observed from comparison with experimental results and complex band structures calculated, a strong dependence of transmission through the crystal on the lattice parameter of the resonators. They found that evanescent waves in the waveguide play a important role when the lattice parameter is in the subwavelength range. Wang et al. [73] investigated the propagation of elastic waves in locally resonant viscoelastic PCs. They found two different types of locally resonant band gaps for quasi-longitudinal and quasi-shear waves. They reported that all changes in the complex band structure and transmission spectra are exclusively due to the dispersive and dissipative effects of viscosity. In addition, negative mass density property may also disappear when viscosity is introduced [73].

All studies about MPCs focused only on propagating waves and did not investigate the evanescent waves and the complex band structure. The main purpose of this study is to investigate the complex band structure, also known as dispersion diagram, of a MPC composed by $BaTiO_3$-$CoFe_2O_4$ inclusions in a polymeric matrix. We consider wave propagation in the *xy* plane (longitudinal-transverse vibration, XY modes, and transverse vibration, Z modes) in an inhomogeneous transversely isotropic elastic solid. The MPC has two-dimensional periodicity and different inclusion geometries - circular, square and rotated square with a 45° angle of rotation with respect to the *x* and *y* axes in a square lattice. Improved plane wave expansion (IPWE) [79,80] and extended plane wave expansion (EPWE) [81–83,10,67–69]formulations are developed in order to obtain the complex band structure for the MPC.

The influence of piezoelectricity and piezomagnetism on the MPC band structure is investigated. Complete band gaps between XY and Z modes are observed for all types of inclusions. To the best of our knowledge, this is the first study analysing the complex band structure and the evanescent Bloch waves in a MPC.

The paper is organized as follows. Section 2 presents IPWE and EPWE approaches for a MPC. In the following, *i.e.*, Section 3, simulated examples are carried out. We analyse first the complete band gaps between XY and Z modes considering just the propagating modes obtained by IPWE method in the first irreducible Brillouin [84] zone (FIBZ). After that, the influence of piezoelectricity and piezomagnetism on Z modes are studied. Next, we obtain the complex band structure and the evanescent modes using EPWE approach. The real part of Bloch wave vector obtained by EPWE is compared to IPWE results. Finally, we investigate the piezoelectricity and piezomagnetism influence on the band gap width as a function of filling fraction. Conclusions are presented in Section 4.

2. Magnetoelectroelastic phononic crystal modelling

This section presents the formulation for a MPC using IPWE, $\omega(\mathbf{k})$, and EPWE, $\mathbf{k}(\omega)$, methods, where $\mathbf{k}$ is the Bloch wave vector, also known as wave number, based on classical elasticity theory. We consider two-dimensional periodicity, *i.e.*, 2D PC, transversely isotropic elastic solid and wave propagation in the *xy* plane.

IPWE and EPWE are semi-analytical methods used to predict the band structure of PCs. IPWE method has the advantage of presenting higher convergence than the traditional PWE method when there is high geometry or material mismatch [80].

The striking issue of using EPWE method is because evanescent modes are obtained naturally and they are not ignored as well as IPWE method. IPWE method considers only the propagating modes, that is to say Bloch wave vector is only real. Furthermore, Bloch wave vector is not restricted to the FIBZ considering EPWE method [10]. Hsue and co-workers [83] proved that evanescent modes obtained by EPWE obey Floquet-Bloch's theorem [85,86].