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Simulation of cross-correlated random field samples from sparse measurements using Bayesian compressive sensing

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ABSTRACT

Cross-correlated random field samples (RFSs) of engineering quantities (e.g., mechanical properties of materials) are often needed for stochastic analysis of structures when cross-correlation between engineering quantities and spatial/temporal auto-correlation of each quantity are considered. Theoretically, cross-correlated RFSs may be simulated using a cross-correlated random field generator with prescribed random field parameters and cross-correlation. In engineering practice, random field parameters and cross-correlation are often unknown, and they need to be estimated from extensive measurements. When the number of measurements is sparse and limited, due to sensor failure, budget limit etc., it is challenging to accurately estimate random field parameters or properly simulate cross-correlated RFSs. This paper aims to address this challenge by developing a cross-correlated random field generator based on Bayesian compressive sampling (BCS) and Karhunen–Loève (KL) expansion. The generator proposed only requires sparse measurements as input, and provides cross-correlated RFSs with a high resolution as output. The cross-correlated RFSs are able to simultaneously characterize the cross-correlation between different quantities and the spatial/temporal auto-correlation for each quantity. The generator proposed is illustrated using numerical examples. The results show that proposed generator performs reasonably well.

1. Introduction

Engineering quantities often exhibit both cross-correlation and auto-correlation over space or time, such as seismic ground motions (e.g., [1]), mechanical properties of soils (e.g., [2,3]), multiscale material properties (e.g., [4,5]), wind fields (e.g., [6]), among others. The cross-correlation between different engineering quantities means that, when one quantity increases, its positively (or negatively) cross-correlated quantity tends to increase (or decreases) (e.g., [7]). On the other hand, auto-correlation means that values of a quantity at neighboring locations are expected to be very similar among each other, and expected to be significantly different if their locations are far away among each other (e.g., [7]). Both the cross-correlation and auto-correlation of engineering quantities greatly affect stochastic analysis of engineering structures, as reported in literature (e.g., [2,3,8–13]). To explicitly and simultaneously model the cross-correlation and auto-correlation, cross-correlated random field samples (RFSs) are often used as input in stochastic analysis of engineering structures.

Cross-correlated RFSs may be simulated using a cross-correlated random field generator with prescribed random field parameters and cross-correlation. Several generators are available in literature. For example, Yamazaki and Shinozuka

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[14] used a covariance decomposition method with statistical preconditioning technique to simulate multivariate processes. Robin et al. [15] proposed to use direct Fourier transform for generating cross-correlated random field. Vořechovský [16] and Cho et al. [17] simulated correlated random fields using Karhunen–Loève (KL) expansion. Zhu et al. [18] proposed to simulate cross-correlated random fields through Cholesky decomposition and conditional sampling based on the joint distribution constructed using copula theory. Note that, for the KL-based cross-correlated random field generator (e.g., [16]), it is often assumed that the random field of each quantity shares the same auto-correlation structure, and that the cross-correlation structure between different quantities is simplified as a cross-correlation coefficient. This assumption facilitates eigen-decomposition of the correlation structures when using KL expansion and simplifies generation of cross-correlated RFSs.

To generate cross-correlated RFSs, all these generators above require prescribed cross-correlation and random field parameters, e.g., the type of auto-correlation function and correlation length. In engineering practice, these parameters are often unknown, and they need to be estimated from measurements. This often requires extensive measurements (e.g., [19,20]), which might not be available due to sensor failure, budget limit etc. (e.g., [21]). When the measurement data are sparse and limited, the parameters estimated and subsequently used in the simulation of cross-correlated RFSs contain significant uncertainty. The effect of such uncertainty on the simulation of cross-correlated RFSs has not been considered in random field generation, except a recent preliminary work by Zhang et al. [22] on quantification of the uncertainty in the estimated power spectrum for random field modelling of a single engineering quantity. In addition, the correlation structure estimated from measurements on one quantity may be different from that of another quantity. This leads to violation of the assumption adopted in the KL-based cross-correlated random field generator (e.g., [16]). Therefore, it is challenging to properly generate cross-correlated RFSs for stochastic analysis of structures when only sparse measurements are available.

This paper aims to address this challenge by developing a novel cross-correlated random field generator using Bayesian compressive sensing or sampling (BCS) (e.g., [23–25]) to simulate cross-correlated RFSs directly from sparse measurements. The proposed generator builds on the KL-based generators (e.g., [16]) and contains a method that is developed to satisfy the assumption of identical correlation structure for all quantities in the KL-based generators. After this introduction, the KL-based generation of cross-correlated random fields and BCS are briefly reviewed. Then a method is proposed to achieve identical correlation structure for all quantities when estimated from measurements. Finally, equations and step by step procedures for the proposed generator are presented. A series of numerical examples are used to illustrate and validate the proposed generator.

2. Simulation of cross-correlated random fields using KL expansion

The use of KL expansion to simulate cross-correlated random fields is briefly reviewed in this section. Consider, for example, two engineering quantities, Q_1 and Q_2. Both Q_1 and Q_2 vary along the same coordinate x. In other words, Q_1 and Q_2 can be modeled as two random vectors: $\boldsymbol{Q_1} = [Q_1(x_1), Q_1(x_2), \ldots, Q_1(x_N)]^{\mathrm{T}}$ and $\boldsymbol{Q_2} = [Q_2(x_1), Q_2(x_2), \ldots, Q_2(x_N)]^{\mathrm{T}}$, respectively. The superscript "T" represents transpose operation. x_i ($i = 1, 2, \ldots, N$) represents locations along one coordinate. When a stationary Gaussian random field is used to model Q_1 or Q_2, $Q_1(x_i)$ or $Q_2(x_i)$ is a Gaussian random variable with a mean of μ_1 or μ_2 and variance σ_1^2 or σ_2^2, respectively, at location x_i ($i = 1, 2, \ldots, N$). Note that $Q_1(x_i)$ is spatially auto-correlated with $Q_1(x_j)$ ($i, j = 1, 2, \ldots, N$), and the auto-correlation is specified by an auto-correlation function with a correlation length λ_{c_1}, or equivalently an auto-correlation matrix $\mathbf{C}_{\mathbf{R}_1}$. Similarly, the correlation matrix for Q_2 is $\mathbf{C}_{\mathbf{R}_2}$ with a correlation length λ_{c_2}. In this study, a bold italic symbol represents a column vector, and a bold upright symbol represents a matrix.

If Q_1 is independent of Q_2, RFSs of Q_1 and Q_2 can be simulated separately by a random field generator, such as a truncated KL expansion (e.g., [26–30]):

$$
\begin{aligned}
\boldsymbol{Q}_1 &\approx \mu_1 \boldsymbol{l} + \mathbf{D}_1 \mathbf{t}_1^P \sqrt{\lambda_1^P} \xi_1^P = \mu_1 \boldsymbol{l} + \mathbf{D}_1 \sum_{i=1}^{P} \boldsymbol{t}_{1_i}^P \sqrt{\lambda_{1_i}^P} \xi_{1_i}^P \\
\boldsymbol{Q}_2 &\approx \mu_2 \boldsymbol{l} + \mathbf{D}_2 \mathbf{t}_2^P \sqrt{\lambda_2^P} \xi_2^P = \mu_2 \boldsymbol{l} + \mathbf{D}_2 \sum_{i=1}^{P} \boldsymbol{t}_{2_i}^P \sqrt{\lambda_{2_i}^P} \xi_{2_i}^P
\end{aligned} \tag{1}
$$

where $\boldsymbol{l}$ is a column vector with all N elements being unity. λ_1^P represents the P (P < N) largest eigenvalues of $\mathbf{C}_{\mathbf{R}_1}$ in a descending order. $\mathbf{t}_1^P$ represents the P eigenvectors of $\mathbf{C}_{\mathbf{R}_1}$, and it corresponds to λ_1^P. Note that although there are N eigenvalues for $\mathbf{C}_{\mathbf{R}_1}$, some of them are near to zero, and $\mathbf{C}_{\mathbf{R}_1}$ can be effectively approximated as $\mathbf{C}_{\mathbf{R}_1} \approx \mathbf{t}_1^P \lambda_1^P (\mathbf{t}_1^P)^{\mathrm{T}}$ (e.g., [27–29,31–36]). ξ_1^P represents P uncorrelated random variables, each of which has a zero mean and a unit variance. $\boldsymbol{t}_{1_i}^P$, $\lambda_{1_i}^P$ and $\xi_{1_i}^P$ represent the i-th column of $\mathbf{t}_1^P$, the i-th diagonal element of λ_1^P, and the i-th element of ξ_1^P. $\mathbf{D}_1$ in Eq. (1) is a diagonal matrix, with all components being the standard deviation (SD) of the Q_1 field. Note that symbols in Eq. (1) with subscript "2" have meaning similar to those with subscript "1", but they are defined for the Q_2 field. In this manner, generation of independent Q_1 and Q_2 RFSs reduces to realizations of P random variables $\xi_{1_i}^P$ and $\xi_{2_i}^P$, such as standard Gaussian random variables. The transformation of a random field/process simulation problem to a problem of simulating uncorrelated random variables by KL expansion also paves the way for stochastic characterization of material properties (e.g., [37,38]), damage quantification and parameter updating in stochastic dynamic systems (e.g., [39–41]), among others.