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Brief paper

# State estimation incorporating infrequent, delayed and integral measurements[☆]

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ABSTRACT

This paper investigates the problem of state estimation incorporating infrequent, delayed and integral measurements. In chemical process, there often exist two types of measurements. On one hand, the measurements for process variables such as flow rates and pressures are sampled frequently and are available nearly instantaneously. On the other hand, the measurements for quality variables such as concentration are sampled infrequently and are available with a delay due to long analysis time involved. Moreover, due to the interval time taken by chemical sample collection, the measurements for some quality variables have another important characteristic: it is a function of the states of the compositions over a period of time. This paper formulates the process with infrequent, delayed and integral measurements as an equivalent variable dimension system, whose measurements are both delay and integration free. Based on the new model, a variable dimension unscented Kalman filter (VD-UKF) is proposed to estimate the states. Furthermore, the stability of the proposed VD-UKF is analyzed. Compared with the existing results, the proposed stability condition is significantly relaxed and the invertibility condition of Jacobian matrices is no longer needed. Finally, a simulation example demonstrates the effectiveness of the proposed method.



## 1. Introduction

In chemical process systems, the measurements for process variables such as flow rates and pressures are usually sampled frequently and are available nearly instantaneously. However, the measurements for certain quality variables such as concentration are sampled infrequently and are only available with considerable time delays. Moreover, instead of depending on a state at certain past instant, the delayed measurements can also be a function of the integral of the states over certain past period of time. This type of measurements is called integral measurements in this paper. For example, in distillation columns, laboratory (lab) analysis is often required for measurement of the distillate and bottoms compositions as the use of online analyzers is often infeasible due to economic considerations or technological difficulty, though online analyzers can be used to measure other process variables such as tray temperatures frequently without time delay (Gopalakrishnan, Kaisare, & Narasimhan, 2011). In order to analyze the composition, a sufficient amount of samples needs to be collected. The time taken to collect the sample is not small compared to the sampling period of the estimator, such that it cannot be ignored. As will be seen in the next section, the compositions of the sample do not represent the compositions at a particular sampling instant but the integral of the compositions within some period of time. In addition, due to offline analysis of the collected samples in a lab the acquired measurement has an unavoidable delay. If the quality variable is inferred directly from the fast-sampled process variable through a model, it can be inaccurate due to model errors, sensor bias or unmodeled disturbances. In such cases the infrequent and delayed integral measurements need to be incorporated into the estimator as the lab analysis is usually more accurate.

By applying multi-rate state estimation techniques, Gudi, Shah, and Gray (1995) designed adaptive strategies to fuse the infrequent and delayed measurements for a fermentation process in a bioreactor. Zambare, Soroush, and Grady (2002) presented real-time implementation of a robust multirate state estimator on a continuous stirred-tank, free-radical, styrene polymerization reactor, where the estimator uses both frequent online measurements

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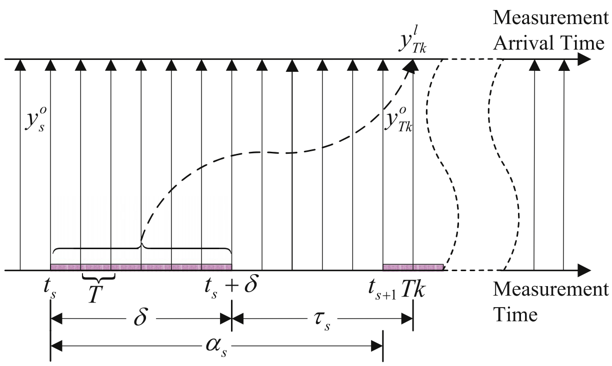


**Fig. 1.** System with online analyzer and lab analysis.

and infrequent offline and delayed measurements. Gopalakrishnan et al. (2011) analyzed several existing methods to incorporate delayed measurements and reinterpret their results under the extended Kalman filter framework. Bavdekar, Prakash, Patwardhan, and Shah (2014) proposed a recursive moving window Bayesian state estimator formulation to utilize delayed measurements to compute the state estimates. All the mentioned papers considered the fact that the measurement for quality variable is infrequent and delayed. However, another important characteristic about such a measurement, the integration nature of sample collection, has not been considered in the literature although the problem is commonly encountered in industries.

This paper investigates the problem of state estimation incorporating infrequent, delayed and integral measurements. First, the mathematical model of the considered process with integral measurements is formulated. Second, for the sake of filter design, true process is reformulated as an equivalent variable dimension system. Then a variable dimension unscented Kalman filter (VD-UKF) is proposed to estimate the states. Furthermore, the stability of the proposed VD-UKF is analyzed. Compared with the existing results (Li & Xia, 2012; Xiong, Zhang, & Chan, 2006), the proposed stability condition is significantly relaxed. The assumption that Jacobian matrices are required to be invertible is no longer needed. This is important since the Jacobian matrices of VD-UKF cannot be always a square matrix; let alone to be invertible. Finally, a simulation example demonstrates the effectiveness of the proposed method.

**Notation.** The notation used in this paper is fairly standard. The superscript "$T$" stands for matrix transposition. diag$\{\cdots\}$ stands for a block-diagonal matrix. $(\cdot)$ denotes the same content as that in the previous parenthesis. Given a real number $x$, $\lceil x \rceil$ denotes the smallest integer greater than or equal to $x$.

## 2. Problem formulation

Consider a continuous, nonlinear process described by the following stochastic differential equation:

$$dx(t) = f_c(x(t), u(t))dt + dw_c(t), \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ is the state of the process; $u \in \mathbb{R}^{n_u}$ is the control input; $f_c$ is the drift function; $w_c(t) \in \mathbb{R}^{n_x}$ is independent Brownian motions with diagonal diffusion matrix $Q_c$.

We consider two types of measurements: fast-rate measurement such as online analyzer and slow-rate measurement such as lab analysis. Similar to Gopalakrishnan et al. (2011), we use the schematic diagram in Fig. 1 to show a process with online analyzer and lab analysis. The online analyzer has frequent and delay-free measurements. Its sampling period is assumed to be $T$. The measurement equation is given by

$$y^o(Tk) = H^o x(Tk) + v^o(Tk), \tag{2}$$

where $v^o(Tk) \in \mathbb{R}^{n_{y^o}}$ is the measurement noise. It is assumed to be normally distributed white noise sequences with zero mean and covariance $R^o$.

The lab analysis is often used as a more accurate measurement of the system quality or critical variables, such as composition or concentration. To get lab data, a technician may need to collect certain amount of chemical samples from the field. Different from the online analyzer, such sample collection process cannot be completed instantaneously but in a time-interval. In addition, the acquired measurement from the lab analysis has an unavoidable delay due to the analysis time of the chemical samples. Thus, the measurement equation is given by

$$y^l(Tk) = H^l(\delta)\int_{t_s}^{t_s+\delta} x(t)dt + v^l(t_s), \quad \text{if } Tk = t_s + \delta + \tau_s, \tag{3}$$

where $s = 1, 2, 3, \ldots$ represents the $s$th lab measurement. $t_s$ is the time instant to start collecting the sample in the field, and $\delta$ is the time interval taken to complete the sample collection. $H^l(\delta)$ is the measurements matrix of the lab analysis. $\tau_s$ is time delay from the time of finishing the sample collection to the completion of lab analysis so that the data becomes available in the distributed control systems (DCS). $v^l(t_s) \in \mathbb{R}^{n_{y^l}}$ is the lab analysis error. It is assumed to be normally distributed white noise sequences with zero mean and covariance matrix $R^l$. The lab analysis is infrequent: the next time to start collecting the sample is $t_{s+1} = t_s + \alpha_s$, i.e., $\alpha_s$ denotes the time interval between two successive starting instants of the lab analysis. The lab analysis can be sampled at irregular intervals and the measurement delay can also vary, that is, both $\alpha_s$ and $\tau_s$ can be time-varying.

Let us further explain such an integral measurement. For simplicity and without loss of generality, we assume that a chemical component concentration is measured by the lab analysis and the state element $x_1$ represents the concentration. That is $x_1(t) = Cx(t)$ with $C = [1 \quad 0 \quad \cdots \quad 0]$. Let $g(t)$ be the flow rate when collecting the samples. In addition, it is assumed that the sample collection starts at time instant $t_s$ and finishes at $t_s + \delta$. Hence, we can calculate the concentration of the collected sample by

$$\frac{\int_{t_s}^{t_s+\delta} g(t)x_1(t)dt}{\int_{t_s}^{t_s+\delta} g(t)dt} = \frac{\int_{t_s}^{t_s+\delta} g(t)Cx(t)dt}{\int_{t_s}^{t_s+\delta} g(t)dt}. \tag{4}$$

In practice, the sample collection rate $g(t)$ can be considered as constant $g$. Under this circumstances, (4) can be simplified as $\frac{C}{\delta}\int_{t_s}^{t_s+\delta} x(t)dt$. Then the measurement of the lab analysis for this sample is $y^l(Tk) = \frac{C}{\delta}\int_{t_s}^{t_s+\delta} x(t)dt + v^l(t_s)$, where $v^l(t_s)$ is the measurement error of the lab analysis. After the lab analysis, the measurement $y^l(Tk)$ becomes available at time $Tk = t_s + \delta + \tau_s$. Let $H^l(\delta) = \frac{C}{\delta}$; then such class of measurements can be written in the general form as shown by Eq. (3).

**Remark 1.** It is worth mentioning that the integral measurement equation in (3) encompasses the traditionally non-integral one as a special case. That is, by letting $H^l(\delta) = \frac{C}{\delta}$ and $\delta \to 0$, the measurement equation (3) will become the traditionally non-integral one:

$$y^l(Tk) = \lim_{\delta \to 0} \frac{C}{\delta}\int_{t_s}^{t_s+\delta} x(t)dt + v^l(t_s) = Cx(t_s) + v^l(t_s).$$

Our problem is how to combine the two types of measurements to estimate the true state of the systems.

## 3. Reformulation of the filtering model

It is difficult to directly use the model (1)–(3) for filtering design, since the measurement $y^l(Tk)$ in (3) not only is delayed but also