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Efficient vibro-acoustic identification of boundary conditions by low-rank parametric model order reduction

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ABSTRACT

A novel method is presented that detects the proper boundary conditions of a test setup in a short time period by combing numerical models with experimental data. This allows for detection and localization of possible anomalies in the assumed boundary conditions of the system. The method works by combining a low-rank parametric model order reduction technique with a model updating strategy, where the boundary conditions of a numerical finite element model are updated by using frequency response function data. This combination makes it possible to update a large amount of parameters, because the assumed low-rank nature of the changes enables the use of non-parametric model order reduction techniques for the calculation of the reduced basis. This is possible, because the system can be rewritten in such a way that the parameter dependencies only show up in the feed-forward matrix of the system, thus no *a priori* sampling of the parameter space is required. Thus, the resulting model can identify a large amount of parameters, including the identification of local changes in the boundary conditions. The method is validated with a test-setup in which an aluminum plate is attached to an acoustic cavity and the boundary conditions are varied gradually, by removing the bolts that are clamping the plate. By applying the proposed model updating scheme to the rotational stiffness along the edge in combination with an additional damping term, it is shown that the proposed method can detect which bolts are removed and also leads to a good match in the frequency response functions. Moreover, it is shown that these results are achieved in only a few minutes, in contrast to the same procedure with full order models.

1. Introduction

The advancement of computer power in the last decades has led to the increased usage of numerical calculation software for predicting the vibro-acoustic behavior of structures, with schemes like the Finite Element Method (FEM) [1] and the Boundary Element Method [2]. When it is desired to use a numerical model in conjunction with measurements, it is important that the numerical model matches the real system to a certain degree. Since it is generally difficult to know all the parameters in the numerical model *a priori*, model updating has risen as a viable tool to iteratively update the numerical model to a required accuracy [3], which usually means that the difference between measured data and the numerical model is minimized. Model updating approaches can be based on mode shapes, eigenfrequencies and frequency response functions

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(FRFs) [3,4] and can work both in the time and frequency domain [5]. The model updating procedure can be used for a wide range of engineering problems, for example boundary detection [6], damage detection [7] and shape identification [8]. Generally, many system evaluations are necessary to obtain the optimal model. Since the used numerical models are usually computationally demanding, it would be beneficial to implement Model Order Reduction (MOR) techniques to speed up the calculations, such as modal reduction [9], balanced truncation [10], proper orthogonal decomposition [11] and techniques based on Krylov subspace projection [12,13]. This paper focuses on Krylov techniques, because of their ability to handle the large, sparse, linear systems that result from vibro-acoustic FE models [14,15], but the proposed method would also work with other projection based model reduction techniques.

When MOR is used for a parameter study, it is desirable to maintain the parameter dependency in either the reduced model, or in the reduced basis used for calculating the Reduced Order Model (ROM). Otherwise, the reduced basis has to be recalculated for every change in parameter values, which requires expensive full system inversions. To avoid this, parametric model order reduction (pMOR) techniques have been developed [16]. Most of these techniques are based on sampling the parameter space, which means that the range of the parameters has to be known *a priori*, and that the accuracy of the reduced model cannot be guaranteed outside of the sampled parameter range. When a large amount of parameters has to be retained, which occurs when strongly localized or spatially distributed parameters have to be detected, and each of these parameters is sampled at several parameter values, the construction of the reduced basis might take too long. Thus in practice the amount of parameters in the ROM is restricted. Recently, it was found by multiple authors [17,18], that under certain low-rank assumptions of the parameter dependence no *a priori* sampling of the parameter space has to be performed to calculate the projection matrix. This can be done by remodeling them as additional localized inputs to the system with extra input vectors that indicate the positions at which the low-rank parameter applies. This means that non-parametric multiple input algorithms for calculating the reduced basis can be used. The result is only a single reduced basis that can be used for all parameter variations. Since no *a priori* sampling has to be applied, a large amount of parameter dependencies can be accounted for by simply increasing the amount of inputs. Also, as has been shown in [19], if the parameters have any correlation in their behavior, which is often the case for dynamic systems, this redundancy can be removed and the size of the ROM stays small, in spite of the many inputs. This methodology has been extended to second order systems in [19].

In this paper model updating is used to determine the structural Boundary Conditions (BCs) of a vibro-acoustic system. While structural BCs are usually modeled as clamped, simply-supported or free, in practice these boundary conditions are never truly satisfied [20]. Since the dynamic behavior of a structure is highly dependent on the BCs, it would be beneficial to have a numerical tool that detects whether the boundary conditions that are assumed in the numerical model match the boundary conditions of the structure and warn the end user about possible discrepancies between the assumed BCs and the actual BCs. The main contribution of this paper is that it derives a methodology that can determine discrepancies and changes in the BCs in a short time frame. The paper is split in two parts: The first part in Sections 2 and 3 describes a generic combined low rank pMOR and model updating procedure, that can efficiently determine the BCs of several vibro-acoustic systems. The described method can handle low-rank changes in the mass, the damping and the stiffness matrix and includes the analytical derivatives, required for the model updating procedure. The second part in Section 4 demonstrates the usage and effectiveness of the method with a specific vibro-acoustic example that utilizes measured FRF data of a clamped plate connected to an acoustic cavity. To establish whether the method can find differences in BCs, one of the edges of the plate is gradually loosened by removing the bolts that are clamping down the plate at the edges. The corresponding rotational stiffness and possible increase in damping of the edge is determined with model updating. It is shown that the method cannot only effectively estimate the areas of reduced stiffness and FRFs, but also that it achieves this in a short time span, as opposed to the same procedure with FRFs of the full model.

2. Low rank parametric model order reduction scheme

To speed-up the model updating process a pMOR scheme is used. The chosen scheme is the second order low-rank scheme as has recently been derived in [19]. This scheme makes it possible to reformulate the parametric system to an equivalent system that can be reduced with non-parametric model reduction techniques. Although the scheme has originally been derived in the time domain, a conversion to the frequency domain is straightforward by using the Laplace transform, as is shown in Section 2.2.

2.1. pMOR formulation in the time domain

As described in [19], the following parametric system description is used:

$$M(r)\ddot{\boldsymbol{x}}(t) + C(q)\dot{\boldsymbol{x}}(t) + K(p)\boldsymbol{x}(t) = \boldsymbol{b}_0 u(t), \tag{1}$$

$$\boldsymbol{y}_0(t) = L_0\boldsymbol{x}(t) + \boldsymbol{d}_0 u(t), \tag{2}$$

in which $M(r) \in \mathbb{R}^{n\times n}$ is the mass matrix, $C(q) \in \mathbb{R}^{n\times n}$ is the damping matrix, $K(p) \in \mathbb{R}^{n\times n}$ is the stiffness matrix, $\boldsymbol{b}_0 \in \mathbb{R}^{n\times 1}$ is the input vector, $L_0 \in \mathbb{R}^{n\times n}$ is the output matrix, $\boldsymbol{x}(t) \in \mathbb{R}^{n\times 1}$ describes the physical state (pressure, displacement, etc.), $u(t)$ is the input signal and $\boldsymbol{d}_0 \in \mathbb{R}^{n\times 1}$ is the feed-forward matrix, which is zero in this case. The parameter dependency in the system matrices $M(r),\ C(q)$ and $K(p)$ is assumed to be affine and of low rank: