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A reliability analysis method based on analytical expressions of the first four moments of the surrogate model of the performance function

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ABSTRACT

For efficiently and accurately analyzing the reliability of structure, the analytical expressions of the first four moments of output are deduced by the Bayesian Monte Carlo method, on which the analytical failure probability can be directly obtained by employing the existing high-order moment standardization technique and the Edgeworth expansion. By use of the proposed procedure, the failure probability of structure can be estimated by an analytical expression without introducing additional errors. Also, only the training points are needed to gain the analytical expressions of the first four moments. Several examples involving numerical test and engineering application are introduced to illustrate the accuracy and the efficiency of the proposed method for reliability analysis of structure.

1. Introduction

In engineering applications, there are always uncertainties, i.e., aleatory uncertainty and epistemic uncertainty, which eventually affect the structural safety. Reliability analysis techniques can measure the safety degree of a structure by taking these uncertainties into consideration. Up to now, many reliability analysis techniques have been proposed. On the one hand, simulation methods, such as Monte Carlo Simulation (MCS) [1], Importance Sampling (IS) [2] and Subset Simulation (SS) [3] are the most used techniques. These methods are accurate and are widely used for various structures involving explicit and implicit performance functions. Note that the computational burden of the MCS method is large although its computational cost is largely reduced with the use of IS and SS.

On the other hand, analytic and semi-analytic methods, such as the First Order Reliability Method (FORM) [4] and the high-order moment method [5,6], etc., are computationally cheaper. FORM approximates the limit state function around the design point based on a Taylor series expansion. In this case, the reliability can be obtained only by the first two moments with a very limited number of performance function evaluations. However, when used with highly non-linear performance functions or multiple non-connected failure domains, FORM may lead to large errors. To alleviate the errors of FORM, a first four-moment-based method is proposed in order to estimate the failure probability using analytical expressions. Generally, if the first four moments of the performance function are accurately estimated, then the failure probability can be expediently obtained with an analytical expression. In order to assess the first four moments of the performance function, several efficient point-estimation methods have been proposed, such as the unscented transformation (UT) technique [7], the multi-

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points estimation method [8], the sparse grid (SG) stochastic collocation method [9], among others. These point-estimation methods use a limited number of characteristic points of the input space and their corresponding weights in order to estimate the probabilistic moments of the performance function. For a complicated performance function, the accuracy of the probabilistic moments can be improved by increasing the number of characteristic points.

Surrogate model methods are other efficient procedures for estimating the reliability of a structure. Generally, the surrogate model method firstly constructs an explicit expression of the performance function, and then the reliability can be estimated by using a MCS-based method on the surrogate model instead of the original performance function. Most of the computational costs of the surrogate model methods are expended in the construction of the surrogate model. Popular surrogate models include the Kriging [10,11], the response surface method [12], the support vector machine [13], the polynomial chaos expansion [14], among others. Generally, these surrogate model methods are efficient and accurate. However, these surrogate model methods need a post-processing computational cost to evaluate the reliability, although the post-processing computational cost is usually ignored because it is relatively smaller than that of evaluating the limit state function, it does still exist. If the analytical solution of the reliability is based on the surrogate model, the post-processing computational cost can be alleviated.

To alleviate the post-processing computational cost of the surrogate model for reliability analysis, an efficient and analytical reliability analysis method is proposed in this paper based on the Bayesian Monte Carlo (BMC) technique [15–17] and the high-order moment method. The BMC technique has been used in previous works for computing definite integrals [15] and reliability analysis [16]. For the estimation of definite integrals [15], the BMC has shown a good balance on the accuracy and the computational cost. The BMC technique for reliability analysis [16] tries to approximate the highly discontinuous indicator function of the failure domain by a Gaussian Process-based meta-model [18]. Actually, the discontinuous indicator function cannot be modeled accurately with limited sampling. Sometimes, the accurate solution of the reliability analysis can be obtained because of the bucking effect [16]. This method has high requirement for selecting the input sample points, i.e., the selected sample points should be located in the vicinity of the most probable failure point. Generally, with the reliability analysis solution, the reliability sensitivity analysis [19–21] can be further obtained.

This work proposes an analytical failure probability solution based on the BMC technique. After the BMC inference stage, the first four moments of the performance function can be analytically expressed uniquely in terms of the training set of the surrogate model; then the failure probability can be obtained with an analytical expression using the high-order moment method. Up to the author's knowledge, this theory has not been previously proposed.

The outline of this paper is as follows: The BMC method is briefly introduced in Section 2. The products of several Gaussian probability density functions are given in Section 3. The analytical expressions of the first four moments are deduced based on BMC in Section 4. The failure probability is analytically expressed in Section 5 by use of the existing HOMST and Edgeworth expansion. The estimation procedure is showed in Section 6. Examples are analyzed in Section 7. Conclusions are provided in Section 8.

2. The Bayesian Monte Carlo method

The Bayesian Monte Carlo is the method which investigates Bayesian alternatives to classical Monte Carlo method. The Bayesian Monte Carlo techniques have been proposed for various engineering applications, such as the estimation of definite integrals [15] and reliability analysis [16]. The BMC approach is capable of offering better performances than crude MCS and even of IS-based variance reduction techniques, both in terms of accuracy and required number of model estimations, by making a more efficient use of the available information. Also, the BMC method can obtain a closed-form analytical expression, which is convenient when the post-processing computational cost required by the estimation algorithms starts affecting the overall computational cost of the failure probability estimation. From a Bayesian perspective, a general estimation problem is turned into an inference problem. More details about BMC can be found in Refs. [15–17]. For the general reliability problem, the relationship between the inputs and the corresponding output can be described by the performance function, $G = g(\mathbf{X})$, where G is the output and $\mathbf{X} = [X_1, X_2, \ldots, X_n]^{\mathrm{T}}$ is the n-dimensional normal random input vector with joint probability density function (PDF) $f_{\mathbf{X}}(\boldsymbol{x})$. The mean vector and covariance matrix for the inputs are $\mathbf{b}$ and $\mathbf{B}$, respectively. If the inputs do not follow the Gaussian assumption, the Rosenblatt transformation [22] can be used to transform the inputs to normal ones.

For estimating the output G, Bayesian inference assumes a prior distribution $f_G(\mathrm{g})$ on $g(\mathbf{X})$ with some observations, i.e., realizations of $g(\boldsymbol{x}^{*(j)})$ at input training samples $\boldsymbol{x}^{*(j)} (j = 1, 2, \ldots, N)$. With the prior distribution $f_G(\mathrm{g})$ and the sample information, the posterior distribution $f_G(\mathrm{g}|\boldsymbol{x}^*)$ can be infered [23]. Assigning a prior over a function is a difficult task. Generally, Gaussian stochastic processes are employed as the prior distributions for $g(\mathbf{X})$ [15]. Under the Gaussian stochastic processes prior assumption, the final Gaussian Process (GP) surrogate model can be obtained based on the Bayesian inference. Actually, there are many surrogate model techniques that can be employed to simulate the output, such as the neural networks, support vector machines, relevance vector machines, extreme learning machines, among other well-known regression and classification techniques in the machine learning field. The neural networks are the mathematical models that imitate the behavioral characteristics brain of the mammals and perform distributed parallel information processing. The support vector machines are supervised learning models, and it can be used to analyze data and identify patterns. More details about surrogate models can be found in Ref. [24]. The prior does not depend on the training samples, but specifies some properties of