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Sparse representations and compressive sampling for enhancing the computational efficiency of the Wiener path integral technique

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ABSTRACT

The computational efficiency of the Wiener path integral (WPI) technique for determining the stochastic response of diverse dynamical systems is enhanced by exploiting recent developments in the area of sparse representations. Specifically, an appropriate basis for expanding the system joint response probability density function (PDF) is utilized. Next, only very few PDF points are determined based on the localization capabilities of the WPI technique. Further, compressive sampling procedures in conjunction with group sparsity concepts and appropriate optimization algorithms are employed for efficiently determining the coefficients of the system response PDF expansion. It is shown that the herein developed enhancement renders the technique capable of treating readily relatively high-dimensional stochastic systems. Two illustrative numerical examples are considered. The first refers to a single-degree-of-freedom Duffing oscillator exhibiting a bimodal response PDF. In the second example, the 20-variate joint response transition PDF of a 10-degree-of-freedom nonlinear structural system under stochastic excitation is determined. Comparisons with pertinent Monte Carlo simulation data demonstrate the accuracy of the enhanced WPI technique.

1. Introduction

Response determination methodologies based on Monte Carlo simulation (MCS) and its variants (e.g., [1,2]) are considered among the most versatile tools in the area of stochastic engineering dynamics. However, for large scale complex systems, these approaches can be computationally prohibitive. Extensive research in the field during the past few decades has shown that alternative approximate analytical and/or numerical schemes offer efficient ways to address a broad class of problems. State-of-the-art semi-analytical techniques for determining the response of stochastic dynamical systems include moments equations and statistical linearization [3–5], stochastic averaging schemes [6], probability density evolution methodologies [7], Fokker-Planck equation solution techniques [8], as well as numerical schemes based on discretized versions of the Chapman-Kolmogorov equation [9–11]. Additional well-established methodologies relate to stochastic reduced order models, stochastic Galerkin and collocation schemes (e.g., [12,13]), as well as techniques based on dynamically orthogonal field equations [14]. Nevertheless, solving high-dimensional nonlinear stochastic differential equations (SDEs) remains a persistent challenge in the field of engineering dynamics.

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One of the recently developed promising techniques in stochastic engineering dynamics relates to the concept of the Wiener path integral (WPI) [15]. Path integral techniques have proven to be potent tools in theoretical physics, with applications ranging from superfluidity to quantum chromodynamics (e.g., [16]). The notion of path integral, which generalizes integral calculus to functionals, was introduced by Wiener [17] and by Feynman [18], independently. Recently, an approximate WPI based technique has been developed for determining the stochastic response of nonlinear and/or hysteretic multi-degree-of-freedom (MDOF) structural systems [19]. The technique exhibits significant versatility and can account even for systems endowed with fractional derivative terms [20]. Furthermore, it has been extended for addressing certain one-dimensional mechanics problems with random material/media properties [21], while preliminary results towards an error quantification analysis can be found in [22]. From a computational efficiency perspective, recent work by Kougioumtzoglou et al. [23] reduced the computational complexity by, potentially, several orders of magnitude as compared to the original formulation and numerical implementation of the technique.

The objective of this paper is to further enhance the computational efficiency of the WPI technique by exploiting recent developments in the area of sparse representations. Indicatively, sparse expansions of multivariate polynomials have been recently used for numerically solving stochastic (partial) differential equations [24–26]. In this paper, compressive sampling procedures are employed in conjunction with group sparsity concepts and appropriate optimization algorithms for decreasing drastically the computational cost associated with determining the system response probability density function (PDF). It is shown that the herein developed enhancement renders the technique capable of treating readily relatively high-dimensional stochastic systems. Two illustrative numerical examples are considered. The first refers to a single-degree-of-freedom Duffing oscillator exhibiting a bimodal response PDF. In the second example, the 20-variate joint response transition PDF of a 10-DOF nonlinear structural system under stochastic excitation is determined. Comparisons with pertinent MCS data demonstrate the accuracy of the enhanced WPI technique.

2. Wiener path integral technique

2.1. Wiener path integral formalism

A wide range of problems in engineering mechanics and dynamics can be described by stochastic equations of the form

$$\boldsymbol{F}[\boldsymbol{x}] = \boldsymbol{w} \tag{1}$$

where $\boldsymbol{F}[.]$ represents an arbitrary nonlinear differential operator; $\boldsymbol{w}$ denotes the external excitation; and $\boldsymbol{x}$ is the system response to be determined. It is noted that Kougioumtzoglou [21] has shown recently that the WPI technique can address not only problems subject to stochastic excitation $\boldsymbol{w}(t)$, but also a certain class of one-dimensional mechanics problems with stochastic media properties; that is, stochasticity is embedded in the operator $\boldsymbol{F}[.]$. Nevertheless, for the purpose of this paper, and without loss of generality, an m-DOF nonlinear dynamical system with stochastic external excitation is considered herein in the form

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} + \boldsymbol{g}(\boldsymbol{x}, \dot{\boldsymbol{x}}) = \boldsymbol{w}(t) \tag{2}$$

where $\boldsymbol{x}$ is the displacement vector process ($\boldsymbol{x}^T = [x_1, \ldots, x_m]$); $\boldsymbol{M}, \boldsymbol{C}, \boldsymbol{K}$ correspond to the $m \times m$ mass, damping and stiffness matrices, respectively; $\boldsymbol{g}(\boldsymbol{x}, \dot{\boldsymbol{x}})$ denotes an arbitrary nonlinear vector function; and $\boldsymbol{w}(t)$ is a white noise stochastic vector process with the power spectrum matrix

$$\boldsymbol{S}_{\boldsymbol{w}} = \begin{bmatrix} S_0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & S_0 \end{bmatrix} \tag{3}$$

Next, relying on the mathematical framework of path integrals [16], the transition PDF $p(\boldsymbol{x}_f, \dot{\boldsymbol{x}}_f, t_f | \boldsymbol{x}_i, \dot{\boldsymbol{x}}_i, t_i)$ can be written as [19]

$$p(\boldsymbol{x}_f, \dot{\boldsymbol{x}}_f, t_f | \boldsymbol{x}_i, \dot{\boldsymbol{x}}_i, t_i) = \int_{\mathcal{C}\{\boldsymbol{x}_i, \dot{\boldsymbol{x}}_i, t_i; \boldsymbol{x}_f, \dot{\boldsymbol{x}}_f, t_f\}} W[\boldsymbol{x}(t)][\mathrm{d}\boldsymbol{x}(t)] \tag{4}$$

with $\{\boldsymbol{x}_i, \dot{\boldsymbol{x}}_i, t_i\}$ denoting the initial state and $\{\boldsymbol{x}_f, \dot{\boldsymbol{x}}_f, t_f\}$ the final state, and $\boldsymbol{x}_i = \boldsymbol{x}(t_i), \boldsymbol{x}_f = \boldsymbol{x}(t_f), \dot{\boldsymbol{x}}_i = \dot{\boldsymbol{x}}(t_i)$ and $\dot{\boldsymbol{x}}_f = \dot{\boldsymbol{x}}(t_f)$. The integral of Eq. (4) represents a functional integration over the space of all possible paths $\mathcal{C}\{\boldsymbol{x}_i, \dot{\boldsymbol{x}}_i, t_i; \boldsymbol{x}_f, \dot{\boldsymbol{x}}_f, t_f\}$, $W[\boldsymbol{x}(t)]$ denotes the probability density functional of the stochastic process in the path space and $[\mathrm{d}\boldsymbol{x}(t)]$ is a functional measure. Further, the probability density functional for the stochastic process $\boldsymbol{x}(t)$ pertaining to the MDOF system of Eq. (2) is defined as (e.g., [19])

$$W[\boldsymbol{x}(t)] = \exp\left(-\int_{t_i}^{t_f} L(\boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}})\mathrm{d}t\right) \tag{5}$$

where $L(\boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}})$ denotes the Lagrangian functional given as