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Nonparametric probabilistic approach of uncertainties with correlated mass and stiffness random matrices

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ABSTRACT

This paper concerns the probabilistic modeling of uncertainties in structural dynamics. For real complex structures, the accurate modeling and identification of uncertainties is challenging due to the large number of involved uncertain parameters. In this context, the nonparametric probabilistic approach which consists in modeling globally the uncertainties by replacing the mass, stiffness and damping reduced matrices by random matrices is attractive since it yields a stochastic modeling for which the level of uncertainties is controlled by a small number of dispersion parameters. In its classical version, these random matrices are assumed to be independent. This assumption is valid (and proven) in absence of information concerning the dependence structure of these random matrices. In some situation, such as the presence of geometry uncertainties, this assumption is not valid any more and may yield an overestimation of the output levels of fluctuation. In this context, the present paper presents an extension of the classical nonparametric probabilistic to take into account a dependence between the random mass and stiffness matrices. This new modeling is illustrated on a beam structure for which the diameter presents spatial random fluctuations along the longitudinal direction.

1. Introduction

For so long time engineers have performed structural analyses using deterministic mathematical models. In especial for many structures the increase of the safety margin was usually employed in the design to support the uncertainties of the physical construction or the variations of the loading or material resistance. In the last years, the development of stochastic models has enhanced the reliability of the dynamic analyses of these structures.

Regardless the chosen strategy uncertainties shall be always considered in structural analyses. There are several methods for including the uncertainties into computational models. Considering probabilistic methods, the classical parametric approach takes into account the uncertainties in the parameters of the computational model [1–3]. This method consists in replacing the uncertain parameters by random variables and then propagate the uncertainty using devoted methods such as the Monte Carlo simulation method. This approach is quite simple to implement and use if there are uncertainty sources with known probability distributions. In cases of real complex structures, this approach becomes inadequate because the sources of uncertainties are not completely known, or they are known but modeled with a large number of hyperparameters that need to be identified.

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The nonparametric probabilistic approach [4,5] is an alternative to overcome the limitations of the parametric one. In this approach, the uncertainties happen at the operator level by modeling the reduced-order generalized mass, stiffness and damping matrices of a structural system as random matrices. There is a small number of dispersion parameters that control the constructed stochastic model and have their experimental identification feasible in an industrial context [6–11]. Furthermore, it is not necessary to know the probability density functions of the model input data. An explicit generator for independent realizations of the random matrices is directly available [4], making its use very practical in the context of a Monte Carlo simulation [12]. The randomness applied directly to the reduced-order generalized matrices allows the solution of the dynamical equations for a chosen amount of vibration modes, which results in a significant reduction of the required computational cost when compared to parametric models.

In its classical form, the nonparametric probabilistic approach assumes that the random mass, stiffness and damping matrices are independent. This assumption completely make sense in case of uncertainties related to the material properties for instance. In absence of information about the dependence of these random matrices, this assumption is correct considering the Information Theory [4]. This assumption yields a conservative stochastic model that will encompass a possible dependence if exists.

In a situation of a structure whose distributed mass has significant influence in its dynamic behaviour, it is clear that uncertainties in chosen geometry parameters promote correlated mass and stiffness uncertainties and consequently, dependent ones. For instance, in most industries (automotive vehicles, aircraft ...), manufacturing tolerances yield uncertainties in the geometry of the structure that will affect the mass and stiffness matrix in a dependent way. In this case, not taking into account the existing dependence structure may overestimate the level of fluctuations of the quantities of interest and in the case of an inverse identification of the dispersion parameters, those later may be underestimated to compensate the non identifiability of the stochastic model.

The objective of this paper is to extend the classical nonparametric probabilistic approach to take into account a correlation/dependency between the mass and stiffness random matrices. To achieve this objective, a global correlation parameter is introduced in addition to the mass and stiffness dispersion parameters. The generator of independent realizations of these pair of random matrices is then constructed. For the proposed construction only one additional parameter is introduced keeping the advantage of the nonparametric probabilistic approach and the classical nonparametric probabilistic approach can be a special case of this new model when setting the correlation parameter to zero.

In this paper, Section 2 presents the nominal model and its related equation of motion. Then Section 3 presents the new extended nonparametric probabilistic approach. Finally, in Section 3, the proposed approach is illustrated and validated through a numerical application consisting of a beam structure with geometry spatial fluctuations.

2. Nominal model

The computational model is constructed using the finite element method in the frequency domain. The structure is analyzed on the frequency range of $0 \leqslant \omega \leqslant \omega_{max}$ and the vector $\boldsymbol{y}(\omega)$ is the $n \times 1$ vector of frequency-dependent displacement amplitudes, where n is the number of degrees of freedom. The equation of motion for the nominal model is

$$(-\omega^2 \mathbb{M} + i\omega \mathbb{D} + \mathbb{K})\boldsymbol{y}(\omega)\, e^{i\omega t} = \mathbb{F}(\omega)\, e^{i\omega t} \tag{1}$$

where $\mathbb{M}, \mathbb{D}$ and $\mathbb{K}$ are the $n \times n$ mass, damping and stiffness matrices, and $\mathbb{F}(\omega)$ is the $n \times 1$ vector of time-independent force amplitudes.

The reduced nominal model is obtained by projecting the nominal model on the subspace spanned by the m first mode shapes. Considering the subscript $k = 1, 2, \ldots, m$ we write the generalized eigenvalue problem as follows

$$\mathbb{K}\boldsymbol{\phi}_k = \lambda_k \mathbb{M}\boldsymbol{\phi}_k \tag{2}$$

The m eigenvalues are indexed as $0 < \lambda_1 \leqslant \ldots \leqslant \lambda_m$ and associated with the corresponding mode shapes $\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \ldots, \boldsymbol{\phi}_m$. Here, only deformation modes are considered for the structure.

We introduce the approximation $\boldsymbol{y}^{(m)}(\omega)$ of $\boldsymbol{y}(\omega)$ written as

$$\boldsymbol{y}^{(m)}(\omega) = \boldsymbol{\Phi}\boldsymbol{q}(\omega) \tag{3}$$

where $\boldsymbol{q}(\omega)$ is the vector of the m generalized coordinates and $\boldsymbol{\Phi}$ is the $n \times m$ modal matrix for the mass-normalized mode shapes.

The reduced nominal model is written as

$$(-\omega^2 \mathcal{M} + i\omega \mathcal{D} + \mathcal{K})\boldsymbol{q}(\omega) = \mathcal{F}(\omega), \tag{4}$$

In Eq. (4), $\mathcal{M} = \boldsymbol{\Phi}^T \mathbb{M} \boldsymbol{\Phi} = I_m$, $\mathcal{D} = \boldsymbol{\Phi}^T \mathbb{D} \boldsymbol{\Phi}$ and $\mathcal{K} = \boldsymbol{\Phi}^T \mathbb{K} \boldsymbol{\Phi} = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_m)$ are the $m \times m$ matrices of generalized mass, damping and stiffness, and $\mathcal{F}(\omega) = \boldsymbol{\Phi}^T \mathbb{F}(\omega)$ is the $m \times 1$ vector of generalized forces.