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Short communication

A faster algorithm for the calculation of the fast spectral correlation

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ABSTRACT

One of the main aims of second order cyclostationary (CS2) analysis is the estimation of the full spectral correlation, allowing the identification of different CS2 components in a signal and their characterisation in terms of both spectral frequency f and cyclic frequency α. Unfortunately, traditional estimators of the full spectral correlation (e.g. averaged cyclic periodogram) are highly computationally expensive and hence their application has been quite limited. On the other hand, fast envelope-based CS2 indicators (e.g. cyclic modulation spectrum, CMS) are bound by a cyclic-spectral form of the uncertainty principle, which limits the extent of the cyclic frequency axis α_{max} at approximately the value chosen for the spectral frequency axis resolution Δf. A recent work has however introduced a ground-breaking approach resulting in a fast algorithm for the calculation of the spectral correlation. This approach is based on the calculation of a series of CMS-like quantities, each scanning a different cyclic-frequency band, given a certain spectral frequency resolution. The superposition of all these quantities allows covering a larger α-band breaking the constraint between maximum cyclic frequency α_{max} and spectral frequency axis resolution Δf, at a limited computational cost. In this paper a new algorithm for the calculation of the same fast spectral correlation is introduced, resulting in a further computational efficiency gain, and a simplification of the computational procedure.

1. Introduction

Second order cyclostationary signals have gained particular attention in the analysis of electrical and mechanical signals in the fields of machines and human health monitoring (for an updated list of applications and related studies the reader can refer to [1]). Pure second order cyclostationary (CS2) signals are fully defined by their instantaneous autocorrelation function (IAS), expected to show a periodic behaviour along its time axis t and a random nature along its time-lag axis τ. For this reason, the estimation of the spectral correlation (SC), defined as the two-dimensional Fourier series-transform of the IAS, is the main objective of CS2 signal analysis [2].

In particular, the SC is a cyclic-spectral quantity, defined as a function of a "spectral" frequency f, resulting from the Fourier transform operation along the time-lag τ, and a "cyclic" frequency α, obtained from the Fourier series operation in the time domain t. This quantity allows a decomposition of the CS2 signal in cyclic-spectral components, each with a continuous

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spectral support band in f and a specific value of cyclic frequency α. The two most established approaches for the estimation of the SC have been the cyclic modulation spectrum (CMS) [3] and the averaged cyclic periodogram (ACP) [4].

The first is probably the most widely spread methodology of cyclic-spectral analysis, and it is based on an envelope-like procedure [5] performed on a discrete short-time Fourier transform (STFT), and a second series of discrete Fourier transform (DFT) operations (one for each frequency of the spectrogram) along the remaining time axis. The advantages of this approach are its simplicity of implementation, its intuitive interpretation, and (most of all) its computational efficiency. The CMS has however a significant drawback: this approach imposes a constraint between the maximum observable cyclic frequency α_{max} and the resolution Δf of the spectral axis, i.e. approximately $|\alpha| < \Delta f$.

The ACP is instead based on the other definition of the SC, i.e. the autocorrelation of the spectrum of the signal. Specifically the ACP is obtained by calculating a discrete-time correlation between the signal's DFT and an α-shifted version of the DFT. This last quantity is in turn obtained by multiplying an α-harmonic complex exponential to the signal prior to the DFT operation. This spectral autocorrelation is obtained separately for different segments of the signal and the results are averaged to ensure lower variance in the indicator. This approach does not suffer from the limitation between α_{max} and Δf, but unfortunately its implementation results in a massive loss of computational efficiency: for signals with lengths between 10–1000 k Samples ACP is 2–6 orders of magnitude slower than CMS. This is mainly due to the massive amount of direct complex multiplications involved in the α shift of the DFT, akin to a traditional (pre-FFT) discrete-time implementation of a Fourier transform.

A recent study [6] has found a quite effective compromise in terms of performance and computational efficiency. The proposed fast spectral correlation (FSC) is based on the calculation of a series of P "scanning spectral correlations" (SSC), akin to the CMS but each having a shifted cyclic-spectral constraint $p\Delta f < \alpha < (p+1)\Delta f$. Then a sum of the $p = 0, \ldots, P-1$ CMS-like SSCs will allow extending the limit for the cyclic frequencies to approximately $|\alpha| < P\Delta f$, with the value of P chosen by the user based on the desired maximum frequency, and limited only by the choice of the overlap between two subsequent windows in the STFT. This methodology substantially allows relaxing the cyclic-spectral constraint ad libitum, and results in a computational cost only approximately P-times the one of the CMS. In a detailed benchmark with previously proposed techniques (i.e. FFT accumulation method [7] and ACP), the FSC has proven to be much faster, i.e. closer to the computational efficiency of the CMS. This advantage, undoubtedly significant, is however diminished when seeking to extend to the maximum the range of cyclic frequencies under analysis, i.e. pushing P towards the maximum (half the window length in samples). In case of typical signal lengths and window sizes this would mean still a computational burden 2–3 order of magnitude greater than the CMS case. Moreover, the complexity of the algorithm, which requires a spectral shift and a spectral correlation for each SSC, could prove a significant obstacle for widespread implementation.

This paper will show how the same FSC can be obtained by means of an even simpler and faster algorithm, very similar to the CMS and only requiring an additional STFT operation. Given the analytical demonstration of identity with the FSC of study [6] and the experimental validation provided in the same study, this paper has been organised as a brief theoretical and algorithmic discussion.

2. The new algorithm and its equivalent with the previous formulation

2.1. Definition

A faster algorithm for the calculation of the FSC is proposed in this section for an L-sample signal $x[n]$ sampled at rate F_s. Given a target maximum cyclic frequency α_{max} and a target spectral resolution Δf (both chosen by the user depending on the nature of the CS2 signal under investigation), an STFT operation is performed on the signal with a window $w[n]$ of length $N_w = F_s/\Delta f$ and with a shift $R = [F_s/(2\alpha_{max})]$ between two subsequent windows (i.e. determining a window overlap of $N_w - R$):

$$X_w[m,k] = \sum_{n=0}^{N_w-1} x[mR+n]w[n]e^{-2j\pi\frac{nk}{N_w}}. \tag{1}$$

As for the previous formulation of the FSC, the selection of the shift R is motivated by the corresponding downsampling of the signal in the STFT, which is characterised by a sampling period R/F_s and will therefore limit the maximum observed cyclic frequency to $\alpha_{max} = F_s/2R$.

In this first operation the n-indexed discrete-time axis is transformed to the k-indexed spectral frequency domain. The main novelty of this approach consists in the second step of the procedure: the calculation of a quantity similar to the STFT of Eq. (1),

$$X_w^P[m,k] = \sum_{n=0}^{N_w-1} x[mR+n]w[n]D_{N_w}^P[n]e^{-2j\pi\frac{nk}{N_w}} \tag{2}$$

where the only additional term $D_{N_w}^P$ corresponds to a Dirichlet kernel function[1]:

[1] Many software libraries and tools provide Dirichlet kernel functions which can be used to compute $D_{N_w}^P[n]$. Each software will likely follow a slightly different definition of the Dirichlet kernel and therefore Eq. (2) must be implemented with care.