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Brief paper

A new approach to finite-time adaptive stabilization of high-order uncertain nonlinear system[☆]

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ABSTRACT

This paper investigates the finite-time stabilization problem for a class of high-order uncertain nonlinear systems. The novel control strategy combining sign function with delicate adaptive technique can handle serious uncertainty and nonlinear growth rate. The convergent time can be adjusted arbitrarily by pre-assigning the design parameter. Finally, a numerical simulation example is given to show the effectiveness of the proposed design method.

1. Introduction

As is well-known, adaptive technique is effective to deal with control problems of nonlinear system with parametric uncertainty, see Krstić, Kanellakopoulos, and Kokotović (1995), Smyshlyaev and Krstić (2010) and references therein. Merged with the adding a power integrator method, this technique enables the stabilization of high-order nonlinear system to achieve rapid progress, see Lin and Qian (2002a,b), Sun and Liu (2007, 2009, 2015) and references therein. On the other hand, due to fast convergence, high tracking precision and disturbance rejection, the study of finite-time control has attracted considerable attention. Bhat and Bernstein (1998, 2000) and Haimo (1986) build up basic finite-time stability theory and the property of settling time function, and these important results accelerate the settlement of the stabilization problem, e.g., Hong and Jiang (2006), Hong, Jiang, and Feng (2010), Hong, Wang, and Cheng (2006), Huang, Lin, and Yang (2005), Li and Qian (2006), Menard, Moulay, and Perruquetti (2010) and Shen and Xia (2008).

In view of existing results being not applicable to finite-time convergence for nonlinear system with parametric uncertainty, high-order nonlinear system is neither feedback linearization at the origin nor affine in the control input. As a result, the finite-time adaptive stabilization of high-order uncertain nonlinear system has been regarded as one of the most challenging issues. Many thanks to Lemma 1 in Hong et al. (2006), which successfully makes the first step to overcome the limitation in theory, a continuous control law is proposed in light of backstepping-like procedure and adaptive idea. However, the requirement on nonlinear function in Hong et al. (2006) is strong, and it is somewhat puzzling to apply Lemma 1 to concrete nonlinear control system. Therefore, an interesting question is put forward spontaneously.

For high-order uncertain nonlinear system, can the restriction of nonlinear function be relaxed in essence, and can the existing results be promoted to solve adaptive finite-time stabilization more conveniently?

It is worth claiming that the affirmative solution to above question is a troublesome task that can be seen from two aspects. (i) The first difficulty is the lack of mathematical tool for adaptive finite-time control. In this paper, an improved Lemma is presented to ensure the boundedness for each possible solution of autonomous system and the finite-time convergence of its component. Please see Remark 3 for a detailed discussion. (ii) A series of obstacles (Remarks 1 and 4–6) emerge in design and analysis owing to the relaxed condition on nonlinear function, that is, powers in the growth rate are allowed to *take values continuously on an interval*. To deal with serious uncertainty and nonlinear growth, sign function and skillful adaptive technique are

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introduced in control design that leads to more intricate design and performance analysis of finite-time controller as well as the constructions of available transformation and Lyapunov function.

Notations. For a vector $x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$, $\bar{x}_i \triangleq [x_1, \ldots, x_i]^T \in \mathbb{R}^i$, $i = 1, \ldots, n$, and we let $\bar{x}_n = x$; the norm $\|x\|$ of $x \in \mathbb{R}^n$ is defined by $\|x\| = \sqrt{\sum_{i=1}^n x_i^2}$; the arguments of functions are sometimes simplified, for instance, a function $f(x(t))$ is denoted by $f(x)$, $f(\cdot)$ or f. For any $y \in \mathbb{R}$, a sign function $\mathrm{sign}(y)$ satisfies: $\mathrm{sign}(y) = 1$ if $y > 0$, $\mathrm{sign}(y) = 0$ if $y = 0$, and $\mathrm{sign}(y) = -1$ if $y < 0$. For a given positive constant a, $\lceil y \rfloor^a \triangleq |y|^a \mathrm{sign}(y)$, $\forall y \in \mathbb{R}$. A continuous function $h : [0, b) \to [0, \infty)$ belongs to class $\mathcal{K}$, if it is strictly increasing and $h(0) = 0$. It belongs to class $\mathcal{K}_\infty$, if $b = \infty$ and $h(y) \to \infty$ as $y \to \infty$.

2. Problem formulation and preliminaries

2.1. Problem formulation

This paper considers the following system:

$$\begin{cases} \dot{x}_i(t) = x_{i+1}^{p_i}(t) + f_i(\bar{x}_{i+1}(t), d), & i = 1, \ldots, n-1, \\ \dot{x}_n(t) = u^{p_n}(t) + f_n(x(t), u(t), d), \end{cases} \tag{1}$$

where $x(t)$ is system state, $u(t) \in \mathbb{R}$ is control input, and $d \in \mathbb{R}^r$ is a parameter vector denoting unknowns. Initial condition is $x(0) = x_0$. For $i = 1, \ldots, n$, $p_i \in \mathbb{R}^{\geq 1}_{odd} \triangleq \{\frac{q_1}{q_2} | q_1$ and q_2 are positive odd integers, and $q_1 \geq q_2\}$ is system high-order, and nonlinear function $f_i(\cdot)$ is continuous with $f_i(0, d) = 0$. It is necessary to point out that asymptotic stabilization problems of system (1) have been investigated in recent years, see Lin and Qian (2002a,b), Sun and Liu (2007, 2009) and references therein, furthermore, Liu and Xie (2011, 2013), Sun, Liu, and Xie (2011), Sun and Liu (2015), Xie and Liu (2012), Zhao and Xie (2014) considered the systems corrupted by time-delay and stochastic noise.

The control objective is to design a continuous adaptive state-feedback controller

$$\begin{cases} u(t) = u(x(t), \hat{\Theta}(t)), & u(0, \hat{\Theta}(t)) = 0, \\ \dot{\hat{\Theta}}(t) = \varphi(x(t), \hat{\Theta}(t)), & \varphi(0, \hat{\Theta}(t)) = 0, \end{cases}$$

where $\varphi(\cdot)$ is continuous, and $\hat{\Theta}(t) \in \mathbb{R}$ is an auxiliary variable to deal with uncertainties, such that the closed-loop state $[x(t), \hat{\Theta}(t)]^T$ is globally uniformly bounded, and $x(t)$ converges to the origin in finite time for any initial condition $[x(0), \hat{\Theta}(0)]^T \in \mathbb{R}^{n+1}$.

The following assumption is needed.

Assumption 1. For each $i = 1, \ldots, n$, there exist an unknown constant $\theta > 0$ and a nonnegative continuous function $b_i : \mathbb{R}^i \to \mathbb{R}$ with $b_i(0) = 0$, such that

$$|f_i(\cdot)| \leq \beta_i |x_{i+1}(t)|^{p_i} + \theta \sum_{j=1}^{i} |x_j(t)|^{\frac{r_i+\omega}{r_j} + \mu_{ij}} b_i(\bar{x}_i(t)),$$

where $0 \leq \beta_i < 1$, $\omega \in (-\frac{1}{\sum_{i=1}^n p_0 \cdots p_{i-1}}, 0)$ with $p_0 = 1$, $\mu_{ij} \geq 0$, $x_{n+1}(t) = u(t)$, $r_1, \ldots, r_{n+1}$ are recursively defined by $r_1 = 1$, $r_j = \frac{r_{j-1}+\omega}{p_{j-1}}$ for $j = 2, \ldots, n+1$.

Remark 1. The power in growth condition defined by $\frac{r_i+\omega}{r_j} + \mu_{ij}$ can take any value on an interval $(0, +\infty)$, which includes the case that all the powers are 1 in Hong et al. (2006). Hence, Assumption 1 enlarges classes of high-order uncertain nonlinear systems by relaxing the restriction of nonlinear function in essence. □

Remark 2. It is of practical importance to achieve global finite-time adaptive stabilization for system (1) under Assumption 1. This point is illustrated by the following example of single-link robot arm with revolute elastic joint Marino and Tomei (1993).

$$\begin{cases} J_1 \ddot{\zeta}_1 + F_1 \dot{\zeta}_1 + N(\zeta_1 - \zeta_2) + Mgl \sin \zeta_1 = 0, \\ J_2 \ddot{\zeta}_2 + F_2 \dot{\zeta}_2 - N(\zeta_1 - \zeta_2) = u. \end{cases} \tag{2}$$

With the help of the coordinate transformation $x_1 = \frac{J_1 J_2}{N} \zeta_1$, $x_2 = \frac{J_1 J_2}{N} \dot{\zeta}_1$, $x_3 = J_2 \zeta_2$, $x_4 = J_2 \dot{\zeta}_2$, system (2) can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3 - \dfrac{F_1}{J_1} x_2 - \dfrac{N}{J_1} x_1 - \dfrac{J_2 Mgl}{N} \sin\left(\dfrac{N}{J_1 J_2} x_1\right) \\ \quad \triangleq x_3 + f_2, \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = u - \dfrac{F_2}{J_2} x_4 - \dfrac{N}{J_2} x_3 + \dfrac{N^2}{J_1 J_2} x_1 \triangleq u + f_4, \end{cases} \tag{3}$$

which satisfies $p_1 = \cdots = p_4 = 1$. Moreover, defining $\theta = \max\{\frac{F_1}{J_1}, \frac{N+Mgl}{J_1}, \frac{F_2}{J_2}, \frac{N}{J_2}, \frac{N^2}{J_1 J_2}\} > 0$ renders $f_2 \leq \theta(|x_1|^{\frac{5}{7}} + |x_2|^{\frac{5}{6}}) b_2$, $f_4 \leq \theta(|x_1|^{\frac{3}{7}} + |x_3|^{\frac{3}{5}} + |x_4|^{\frac{3}{4}}) b_4$. Now, Assumption 1 holds with $f_1 = f_3 = 0$, $b_1 = b_3 = 0$, $b_2 = x_1^{\frac{2}{7}} + |x_2|^{\frac{1}{6}}$, $b_4 = x_1^{\frac{4}{7}} + x_3^{\frac{2}{5}} + |x_4|^{\frac{1}{4}}$, $\mu_{ij} = 0$, $i = 1, \ldots, 4$, $j = 1, \ldots, i$, and $\omega = -\frac{1}{7} \in (-\frac{1}{4}, 0)$. Therefore, without the precise information on the constants F_1 and F_2 that represent the unknown friction coefficients in system (2), the global finite-time adaptive stabilization will be achievable by control strategy in this paper. □

2.2. Preliminaries

To begin with, we provide several key lemmas that play a crucial role in theoretical analysis.

Lemma 1. *For autonomous system $\dot{x}(t) = f(x(t))$, suppose that $x(t)$ is defined on $[0, \infty)$, and $D \subset \mathbb{R}^n$ is a domain containing $x = 0$. Let $W : D \to [0, \infty)$ be a continuously differential function satisfying $W(x) = 0$ if and only if $x_1 = 0$, and $W(x) > 0$, for all $x_1 \neq 0$, where $x(t) = [x_1(t), x_2(t)]^T$. Assume that time derivative of $W(x)$ along the solution of system $\dot{x}(t) = f(x(t))$ satisfies $\dot{W}(x(t)) + cW^\alpha(x(t)) \leq 0$ with $c > 0$ and $0 < \alpha < 1$ being known constants. Then, there exists a finite time $T \geq 0$, such that $x_1(t) = 0$, $\forall t \geq T$, moreover, $T \leq \frac{1}{c(1-\alpha)} W^{1-\alpha}(x(0))$.*

Proof. For the system $\dot{y}(t) = -cy^\alpha(t)$, $y(0) = y_0$, $y(t) \geq 0$, by direct integration, one easily obtains its solution defined as follows:

$$y(t; 0, y_0) = \begin{cases} \left(y_0^{1-\alpha} - c(1-\alpha)t\right)^{\frac{1}{1-\alpha}}, & t < \dfrac{1}{c(1-\alpha)} y_0^{1-\alpha}, \\ 0, & t \geq \dfrac{1}{c(1-\alpha)} y_0^{1-\alpha} \end{cases} \tag{4}$$

for $y_0 \neq 0$, and $y(t; 0, y_0) = 0$ for $y_0 = 0$. Theorem 5.11 in Kartsatos (2005) yields $W(x(t)) \leq y(t; 0, W(x(0)))$, $\forall t \geq 0$. Specifically, in light of (4), it follows that $W(x(t)) \leq y(t; 0, W(x(0))) = 0$ for $t \geq \frac{1}{c(1-\alpha)} W^{1-\alpha}(x(0))$. Since $W(x) = 0$ if and only if $x_1 = 0$, and $W(x) > 0$, for all $x_1 \neq 0$, it is easy to get $x_1(t) = 0$, $t \geq \frac{1}{c(1-\alpha)} W^{1-\alpha}(x(0))$. Hence, $T \leq \frac{1}{c(1-\alpha)} W^{1-\alpha}(x(0))$. □

Remark 3. The finite-time stability of autonomous system $\dot{x}(t) = f(x(t))$ has been achieved by the method in Bhat and Bernstein (2000), Hong et al. (2006), Shen and Xia (2008). However, in contrast to Lemma 1 of this paper, these results are somewhat