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Reduced order surrogate modeling technique for linear dynamic systems

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ABSTRACT

The availability of reduced order models can greatly decrease the computational costs needed for modeling, identification and design of real-world structural systems. However, since these systems are usually employed with some uncertain parameters, the approximant must provide a good accuracy for a range of stochastic parameters variations. The derivation of such reduced order models are addressed in this paper. The proposed method consists of a polynomial chaos expansion (PCE)-based state-space model together with a PCE-based modal dominancy analysis to reduce the model order. To solve the issue of spatial aliasing during mode tracking step, a new correlation metric is utilized. The performance of the presented method is validated through four illustrative benchmarks: a simple mass-spring system with four Degrees Of Freedom (DOF), a 2-DOF system exhibiting a mode veering phenomenon, a 6-DOF system with large parameter space and a cantilever Timoshenko beam resembling large-scale systems.

1. Introduction

Modeling and simulation (M&S) play important roles in the analysis, design, and optimization of engineering systems. M&S targets deriving models with enough accuracy for their intended purpose. These models can be used to predict the systems' response under different loading and environmental conditions. Although the development of computational power of modern computers has been very fast in recent years, more precise description of model properties and more detailed representation of the system geometry still result in large-scale, more complex models of dynamical systems with considerable execution time and memory usage. Model reduction [1], efficient simulation [2–4] and parallel simulation methods [5,6] are different strategies to address this issue.

Nowadays, due to an ever increasing need for accurate and robust prediction of systems' response, engineering models are almost always employed with some parameters to allow variations in material properties, geometries, initial and boundary conditions. Consequently, numerous model evaluations are normally required for the procedure of modeling, identification and design of real-world applications in order to optimize the models' performance. This can be prohibitively expensive for computationally demanding models. In the literature, several approaches address this issue by replacing such models with approximations that can reproduce the essential features faster, *e.g.* surrogate modeling [7] and parametric model order reduction (PMOR) [8].

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Surrogate models can be created intrusively or non-intrusively. In intrusive approaches, the equations of a system are modified such that one explicit function relates the stochastic properties of the system responses to the random inputs. The perturbation method [9] is a classical tool used for this purpose but it is only accurate when the random inputs have small coefficients of variation (COV). An alternative method is intrusive Polynomial Chaos Expansion (PCE) [10]. It was first introduced for Gaussian input random variables [11] and then extended to the other types of random variables leading to generalized polynomial chaos [12,13]. In non-intrusive approaches, already existing deterministic codes are evaluated at several sample points selected over the parameter space. This selection depends on the methods employed to build the surrogate model, namely regression [14,15] or projection methods [16,17]. Kriging [18,19] and non-intrusive PCE [20] or combination thereof [21,22] are examples of the non-intrusive approaches. The major drawback of PCE methods, both intrusive and non-intrusive, is the large number of unknown coefficients in problems with large parameter spaces, which is referred to as the curse of dimensionality [23]. Sparse [24] and adaptive sparse [20] polynomial chaos expansions have been developed to tackle this issue.

Parametric model order reduction (PMOR) methods are analogous to regression-based non-intrusive surrogate modeling except that in PMOR, interpolations are performed not only on system responses [25], like surrogate modeling [26], but also on the reduced bases [27] and reduced models [28,29] evaluated over the parameter space. In this regard, some attempts have been made recently to surrogate the models as well, *e.g.* the modal models [30,31], NARX models [32] and time-varying ARMA models [33]. However, they were only applied to very small cases with single excitation and single sensor location, known as single-input, single-output (SISO) systems. Since the extension of the modal models to nonlinear systems is not straightforward and NARX models become very complex for structures with Multi-input, Multi-output (MIMO), they are not a suitable model for real life applications.

State-space (SS) is the most common model for dynamic systems due to the following reasons: (i) it can represent MIMO systems as convenient as SISO systems, (ii) it can easily be expanded to the more complex physical phenomena such as nonlinear systems, and time-varying systems. Unlike PMOR methods which are mostly developed for SS models [8], there is a lack of literature in surrogating these models. Moreover, complex dynamic systems have large model orders, therefore applying PCE directly on these models may not be feasible. An appropriate approach could be to construct surrogate models jointly with a proper model reduction method which is at the focus of this paper. It opens up a new field which requires more investigation.

In [34], Yang et al. combine intrusive PCE with a projection based model reduction method and applied them to uncertain structures. In [35], Kim developed a reduced order modeling technique for parametric linear systems. In this method, Karhunen-Loeve approach is used in frequency domain to calculate optimal modes subject to the simultaneous excitations. The method has been successfully applied to a structural systems and later in [36], was extended to aeroelastic systems.

The main contribution of this paper is developing a reduced-order PCE-based surrogate model for linear dynamic systems when they are presented in SS form. For this purpose, the PCE-based surrogate model is developed in conjunction with modal dominancy analysis [37] in order to reduce the model order, if required, while preserving the model structure. Computational efficiency is another beneficial feature of the modal dominancy analysis which is of our special interest in this work. The proposed method is organized around two steps. In the first step, the modes are transformed to have similar orientations to those of a reference model and in the second step, interpolation is performed using PCE. The other novelties of the paper are: (i) new correlation metric to track system modes when small number of sensors are available, (ii): *QR*-factorization is employed to treat the well-known problem of non-unique eigenvectors of coalescent modes.

The outline of the paper is as follows. In Section 2, the problem is formulated and in Section 4, after a short review of the pertinent materials from linear system theories and polynomial chaos expansion, the proposed methodology is elaborated. In Section 5, the method is applied to several academic case studies resembling challenges in the structures.

2. Problem formulation

Consider the spatially-discretized governing second-order equation of motion of a linear time-invariant (LTI) system as

$$\begin{aligned} \boldsymbol{M}(\boldsymbol{x})\ddot{\boldsymbol{q}}(t) + \boldsymbol{V}(\boldsymbol{x})\dot{\boldsymbol{q}}(t) + \boldsymbol{K}(\boldsymbol{x})\boldsymbol{q}(t) &= \boldsymbol{f}(t) \\ \boldsymbol{y}(t) &= \boldsymbol{C}'(\boldsymbol{x})\boldsymbol{q}(t) \end{aligned} \tag{1}$$

in which for an N-DOF system with n_u system inputs and n_y system outputs, $\boldsymbol{q}(t) \in \mathbb{R}^N$ is the displacement vector, $\boldsymbol{y}(t) \in \mathbb{R}^{n_y}$ is the system output, $\boldsymbol{f}(t)$ is the external load vector which is governed by a transformation of stimuli vector $\boldsymbol{f}(t) = \boldsymbol{P_u u}(t)$; with $\boldsymbol{u}(t) \in \mathbb{R}^{n_u}$. $\boldsymbol{x} \in \mathbb{R}^{n_x}$ is the parameter vector and real positive-definite symmetric matrices $\boldsymbol{M},\ \boldsymbol{V},\ \boldsymbol{K} \in \mathbb{R}^{N\times N}$ are mass, damping and stiffness matrices, respectively. The output matrix $\boldsymbol{C}' \in \mathbb{C}^{n_y\times N}$, maps the displacement vector to the output $\boldsymbol{y}(t)$.

The SS realization of the equation of motion in Eq. (1) can be written as

$$\begin{aligned} \dot{\boldsymbol{\eta}}(t) &= \boldsymbol{A}(\boldsymbol{x})\boldsymbol{\eta}(t) + \boldsymbol{B}(\boldsymbol{x})\boldsymbol{u}(t) \\ \boldsymbol{y}(t) &= \boldsymbol{C}(\boldsymbol{x})\boldsymbol{\eta}(t) + \boldsymbol{D}(\boldsymbol{x})\boldsymbol{u}(t) \end{aligned} \tag{2}$$

where $\boldsymbol{A} \in \mathbb{C}^{2N\times 2N}$, $\boldsymbol{B} \in \mathbb{C}^{2N\times n_u}$, $\boldsymbol{C} \in \mathbb{C}^{n_y\times 2N}$, and $\boldsymbol{D} \in \mathbb{C}^{n_y\times n_u}$. $\boldsymbol{\eta}(t) = [\boldsymbol{q}(t)^T, \dot{\boldsymbol{q}}^T(t)]^T \in \mathbb{R}^{2N}$ is the state vector. $\boldsymbol{A}$ and $\boldsymbol{B}$ are related to mass, damping and stiffness as follows