Mechanical Systems and Signal Processing 111 (2018) 194–209

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Robustness of stochastic expansions for the stability of uncertain nonlinear dynamical systems – Application to brake squeal

L. Nechak [a,*], S. Besset [a], J.-J. Sinou [a,b]

[a] *Laboratoire de Tribologie et Dynamique des Systèmes, UMR CNRS 5513, École Centrale de Lyon, 36 avenue Guy de Collongue, 69134 Écully Cedex, France*
[b] *Institut Universitaire de France, 75005 Paris, France*

ARTICLE INFO

Article history:
Received 19 October 2017
Received in revised form 14 December 2017
Accepted 15 January 2018

Keywords:
Nonlinear dynamical systems
Stability
Uncertainty
Random parameter
Meta-models
Generalized polynomial chaos
Wiener-Haar expansion
Brake squeal

ABSTRACT

This paper is devoted to the prediction and analysis of brake squeal under random uncertainty. This problem, which is a particular application of a wider issue, namely the stability of random parameter-dependent (RPD) nonlinear dynamical systems, is undertaken by using the non-intrusive generalized polynomial chaos (GPC) and Wiener-Haar expansions. The main objective is to assess the capacities of these meta-models within this framework. A reduced nonlinear non-equally damped, iso-damped and non-damped, disc/pad models are considered in this perspective in order to analyze the robustness of the proposed meta-models with respect to perfect and non-perfect mode coalescence. It turns out that the Wiener-Haar meta-model shows a more robust performance than GPC expansion and consequently offers a more reliable tool for the nonlinear stability analysis and thus for the prediction of brake squeal under parameter uncertainty.

1. Introduction

The duty to respect the increasingly restrictive acoustic standards imposes a better understanding and control of the mechanisms generating noises, in particular the brake squeal [1–3]. In this context, the implementation of a suitable strategy for the squeal mitigating requires a high capacity to efficiently predict it. The latter issue which is closely linked to the stability problem of nonlinear dynamical systems, is not trivial because of the nonlinear and the dispersive characters of squeal and the high dimension of models describing it. The nonlinear character is mainly related to the nonlinear nature of the contact and frictional forces involved in the mechanisms generating squeal [4,5] while the dispersive character is induced by the non-negligible sensitivity of squeal towards design parameters as shown in numerous studies [6–9]. Hence the taking into account of uncertainty for a more reliable and robust predicting of squeal has become a strong require as demonstrated by the experimental and numerical results in [10,11]. Moreover, it is necessary to propose representative models often based on finite element models (FEMs) (with high numbers of degrees of freedom of the sub-structures and their contact interfaces) in order to faithfully reproduce the squeal propensity [12,13]. So taking into account the presence of uncertainties for nonlinear mechanical systems of high dimension make the prediction of squeal computationally prohibitive and so, a challenging issue.

* Corresponding author.
E-mail addresses: lyes.nechak@ec-lyon.fr (L. Nechak), sebastien.besset@ec-lyon.fr (S. Besset), jean-jacques.sinou@ec-lyon.fr (J.-J. Sinou).

https://doi.org/10.1016/j.ymssp.2018.01.021
0888-3270/

Model reduction and/or meta-modelling are two areas exploited to enhance the efficiency of squeal prediction methods. The latter are based either on the complex eigenvalues analysis (CEA) [14] or on the nonlinear transient analysis methods [15]. Concerning the meta-modelling, this area searches for a surrogate model which replaces the high dimensional model by some black-box which is then completely defined by using a finite number of input/output simulations of the original model [16].

Numerous methods have been developed in order to enhance the efficiency of squeal prediction under parameter uncertainty. The fuzzy approach was proposed,for instance, in [17] for the analysis of the squeal occurrence in a industrial brake system with a friction coefficient and a contact pressure described by fuzzy numbers. A more sophisticated strategy based on Control approach, Homotopy and Component mode synthesis (CHOC), was proposed in [18] for the prediction of friction induced instability in a pad/disc system under parameter variations. In this method, a fuzzy based control approach is defined for managing the sliding contact between the disc and the pad while Homotopy perturbation and projection techniques are carried out for the treating of the uncertainties. In another register, kriging meta-models were proposed in [19,9] for the prediction of parameter-dependent occurrences of brake squeal. These meta-models that are built by exploiting spatial correlation functions and a finite number of input/output simulations based on FEMs, have shown an interesting efficiency with respect to the classical parametric approach. In the same context, the response surface method (RSM) was proposed in [20] for the approximating of unstable modes of a disc brake system. The main objective in this study was to approximate the parameter-dependent linear stability of the disc brake system by a meta-model which can then be used for the optimizing of the stability behavior of the system. Otherwise, probabilistic meta-model based on the generalized polynomial chaos (GPC) expansion was previously proposed for the approximating of random eigenvalues characterizing the stability of a linear pad/disc system [21]. This meta-model was calculated intrusively: the random eigenvalue problem associated to the FEM of the considered linear pad/disc system is projected onto the GPC space, giving rise to an augmented deterministic eigenvalue problem the solution of which enables to determine the coefficients of the GPC meta-model. This meta-model was shown to be a promising alternative to the MC method, the latter having been proposed in a previous work [22] to study the probability of squeal occurrence through a sub-structured linear FEM of a simplified brake system. In the same field, brake squeal was analyzed by using the evidence theory in order to tackle the cases where uncertainties are related to data imprecision induced by a lack or conflict information [23]. Otherwise, different approaches aiming to predict brake squeal under hybrid uncertainties were proposed these last years. In [24], interval and random variables were simultaneously considered to describe the friction coefficient and Young modulus, and the brake pressure, density of component materials, and thickness of back plate respectively in order to analyse the stability of a linear disc brake system by using the RSM combined with the CEA method. The same study by considering random and fuzzy uncertainties was also performed and presented by the same authors in [25].

This study focuses on the prediction and analysis of random friction-dependent occurrence of squeal by using probabilistic meta-models. More particularly, the non-intrusive GPC and Wiener-Haar expansions [26] recently developed for the stability analysis [27] and the estimating of the probability density function of random friction-induced limit cycle oscillations [28] in uncertain friction systems with small numbers of degrees of freedom, are considered in this paper. The common advantage of both meta-models is related to their non-intrusive implementation which permits to take into account uncertainty of the nonlinear static equilibrium when dealing with the stability analysis of nonlinear dynamical systems with random parameters. Moreover, no operation is required on the original FEM by opposite to the intrusive technique used in [21]. However, only a finite number of FEM based simulations are necessary. The main objective of this study is then to assess the capacities of both the GPC and Wiener-Haar expansions to be efficiently used for the stability analysis of RPD- nonlinear systems and more particularly for the prediction and analysis of squeal occurrence under random uncertainty. In this perspective, a nonlinear pad/disc system is considered. More particularly, this study investigates the robustness of the two proposed meta-models (GPC and Wiener-Haar expansions) with respect to perfect and non-perfect mode coalescence by considering non-equally damped, non-damped and iso-damped configurations for the used pad/disc system. Considering these three cases is justified by demonstrating the effectiveness of the developed strategy for more or less complex cases in terms of solution approximation and associated meta-modelling. Indeed one of the main limitations of various approximation methods for predicting the friction-dependent eigenvalues is the need to use high degrees to accurately approximate quantities with strong variations [21]. This situation occurs, for example when one is in the case of a perfect coalescence patterns (which occurs at the critical bifurcation for non-damped or iso-damped systems, see for more details [29]). As previously explained by Sarrouy et al. [21], in these two specific configurations, the rst derivative of some eigenvalue may not be continuous over the input range of interest (more precisely at the critical bifurcation point) resulting in a very poorly polynomial approximation to describe the eigenvalue evolution. Moreover considering both the equally and non-equally damped coalescences will allow to show the differences in the performance of the GPC and Wiener-Haar expansions in each case and also to demonstrate the potential of the proposed methods. In fact, there are numerous studies that have demonstrated the effect of damping on the stability of brake systems with the well-known shifting or smoothing effect on the coalescence curves and the destabilizing paradox (see for more detail the following contributions [29–32]). The proposed study is therefore not intended to re-examine what has been done on the subject but only to determine the influence of damping on the performances of the GPC and Wiener-Haar expansions within the stability analysis framework. In other words, the main objective of this paper is to address the use and the efficiency of the GPC and Wiener-Haar expansions in the general context of stability of RPD-nonlinear dynamical systems, in particular for mechanical system subjected to friction-induced vibration with perfect or non-perfect coalescence patterns for the perspective of brake squeal prediction under parameter uncertainty.