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Identification of the viscoelastic response and nonlinear damping of a rubber plate in nonlinear vibration regime

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ABSTRACT

Three different dissipation models have been used to identify the increase of damping with the vibration amplitude for a rubber rectangular plate. For this purpose, a square rubber plate made of silicone with fixed edges has been tested and its linear and nonlinear responses have been measured by laser Doppler vibrometers. First, a reduced-order model, using energy based approach and global discretization, has been constructed, taking into account geometric imperfections; the linear viscous damping at each excitation level in the nonlinear regime has been identified from the experimental data. This numerical model with linear viscous damping has been widely validated and constitutes the basis for comparison with subsequent damping identifications. Then, three different single degree of freedom models have been fitted to the same experimental data; each model has a different damping description. Specifically, the models are based on a modified Duffing oscillators with linear, quadratic and cubic stiffness and: (i) a linear viscous damping; (ii) a nonlinear viscoelastic dissipation described by the loss factor; (iii) a standard linear solid viscoelastic model with nonlinear springs. The dissipation identified by the different models is discussed and confirms the major nonlinear nature of damping as a function of the vibration amplitude.

1. Introduction

Understanding the structural damping is of critical importance for effective design. The importance increases manifolds when the structure experiences large amplitude vibrations, as its nonlinear dynamics is even more influenced on the amount of damping than in the linear (i.e. small amplitude) vibration regime [1]. Experimental data is mandatory to identify the damping present in a structure, both in linear and nonlinear regime. However, damping cannot be directly measured, so a dissipation model must be introduced in order to identify its value. Dissipation depends, among others, on the geometry, material, surrounding fluid, boundary conditions and vibration mode shape.

There are well established tools available to extract the viscous damping ratio in the linear vibration regime. Experimental modal analysis is one among them and has become the industrial standard decades ago. However, thin walled structures such as plates, panels and shells experience large amplitude vibrations, i.e. vibration amplitude of the order of the thickness giving rise to geometrical nonlinearity, during their normal operating conditions and experimental modal analysis or any

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other tools based on the linear vibration assumption cannot be used to extract the damping of them. Hence there is a clear need for developing tools to extract damping in the nonlinear vibration regime for thin walled structures.

Recent experimental studies show that the damping present in a structure increases nonlinearly as the vibration amplitude increases. This phenomenon is well documented in cantilever beams [2], plates [3,4], panels [3] and shells [5] considering viscous damping model for representing the dissipation energy.

In order to predict the nonlinear dynamics of a structure using a reasonable amount of degrees of freedom, with intrinsic numerical advantages in reliability and computational cost, it is necessary to build a Reduced Order Model (ROM) [6,7]. Often the damping in the ROM is introduced as linear viscous damping, e.g. by using Rayleigh's dissipation function [7]. Developing an accurate ROM for a structure involves numerous complexities. A simple tool to extract the damping of a structure from its experimentally measured nonlinear response, without the need of developing a ROM, would give engineers/scientists clear advantage. To address this need, a tool based on harmonic balance method was developed considering viscous damping model in a modified Duffing nonlinear resonator with quadratic and cubic stiffness terms [8]. This tool successfully extracts the damping ratio present in a structure from its amplitude-response curves at different excitation levels and the results match very well with the damping identified from the sophisticated ROM of the structure. Experimental results show that the increase in damping with the peak vibration amplitude (and the level of harmonic force excitation) is substantially large [8]. Not taking it into account would lead to inefficient design of such structures and very large overprediction of the vibration amplitude. In fact, most of the studies in nonlinear vibration show complicated nonlinear dynamics which is fully destroyed by the increase of the actual damping due to the large amplitude of vibrations.

Structures made of rubber-like or biological materials exhibit substantial viscoelastic behaviour [9]. For these structures, the viscous damping model does not capture accurately the dissipated energy. Viscoelastic damping models such as the Kelvin-Voigt, hysteretic, standard linear solid or Boltzmann models have been used for that purpose [10–13]. A previous study by our research group shows the variation of relaxation time (dissipation parameter of Kelvin-Voigt model viscoelastic model) versus the vibration amplitude of rubber plates [14]. The nonlinear damping introduced by the Kelvin-Voigt model is not sufficient to capture the damping exhibited by the structure during large amplitude vibrations. So, the relaxation time has to be increased as the vibration amplitude increases to match the numerical response with experimental measurements, implying that Kelvin-Voigt model is not sufficient for modelling the nonlinear damping.

Hysteretic damping (where stiffness and damping are represented together by a complex spring) has been used extensively to describe the dissipation present in viscoelastic systems using the loss tangent. For example, the loss tangent of an aluminium plate having viscoelastic core with various fiber orientation was identified experimentally by Berthelot et al. [15]. A novel method to identify the loss tangent in the linear regime was developed by Liu and Ewing [16]. The loss and storage energies were experimentally calculated by dividing the mobility at the driving point by the mobility of the measurement point. The loss tangent was also identified by frequency response functions measured on different points of the geometry for a composite plate [17]. However, the variation of hysteretic damping during large-amplitude vibrations of a structure has not been completely addressed yet, as per authors' knowledge.

It is evident that a nonlinear dissipation model is necessary to predict structural dynamics at large-amplitude vibration [3]. Few researchers have attempted to use phenomenological nonlinear damping for various structures. Zaitsev et al. [18] used the Kelvin-Voigt model for describing the nonlinear damping of micromechanical oscillator and concluded that the model is better than linear viscous damping but not sufficient and further work is required. Gottlieb and Habib [19] used a phenomenological nonlinear damping model to understand the large amplitude, quasi-periodic and chaotic vibrations of a spherical pendulum. Eichler et. al. [20] used a damping model containing a nonlinear term proportional to the square of the vibration amplitude multiplied by the velocity, similar to the one previously introduced in [18], without any derivation; it was applied to estimate the damping in carbon nanotubes and graphene devices. Recently Amabili [21] derived a model of nonlinear damping based on a fractional standard linear solid material after introducing geometric nonlinearity in it. The model was successfully compared to experimental results for vibrations of a plate, a beam and a curved panel in geometrically nonlinear regime.

In the present study, three different dissipation models have been used to identify the increase of damping with the vibration amplitude for a rubber rectangular plate. An overview of the organization of the study with the different models developed is shown in Fig. 1. For this purpose, a square rubber plate made of silicone with fixed edges has been tested and its linear and nonlinear responses have been measured by laser Doppler vibrometers. First, a ROM, using energy based approach and global discretization, has been constructed, taking into account geometric imperfections; the linear viscous damping at each excitation level in the nonlinear regime has been identified from the experimental data. This numerical model with linear viscous damping has been widely validated and constitutes the basis for comparison with subsequent damping identifications. Then, three different single degree of freedom (SDOF) models have been fitted to the same experimental data; each model has a different damping description. Specifically, the models are based on a modified Duffing oscillators with linear, quadratic and cubic stiffness and: (i) a linear viscous damping; (ii) a nonlinear viscoelastic dissipation described by the loss factor; (iii) a standard linear solid viscoelastic model with nonlinear springs. The dissipation identified by the different models is discussed and confirms the major nonlinear nature of damping as a function of the vibration amplitude.