Automatica 58 (2015) 183–190

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Output feedback regulation of a chain of integrators with unknown time-varying delays in states and input[☆]

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ARTICLE INFO

Article history:
Received 8 October 2014
Received in revised form
25 February 2015
Accepted 11 May 2015
Available online 7 June 2015

Keywords:
Global regulation
Chain of integrators
Observer
Unknown time-varying delays

ABSTRACT

We consider a problem of global asymptotic regulation of a chain of integrators that has unknown time-varying delays in both states and input by output feedback. The time-varying delays are only known to be bounded, and their bounds and time-varying rates are unknown. To solve the considered problem, we introduce a newly designed adaptive output feedback controller. For system analysis, we give a new transformation and techniques in order to deal with new phenomena associated with unknown time-varying delays in states.

1. Introduction and problem formulation

Time-varying delays problems in control systems have received much attention and currently remain as active research areas in control engineering field (Bekiaris-Liberis, Jankovic, & Krstic, 2012; Bekiaris-Liberis & Krstic, 2010, 2012, 2013; Bresch-Pietri, Chauvbin, & Petit, 2012; Choi & Lim, 2006, 2010; Gielen, Teel, & Lazar, 2013; Jankovic, 2010; Karafyllis, 2006; Koo, Choi, & Lim, 2012; Krstic, 2010; Lei & Lin, 2007; Lin & Fang, 2007; Polyakov, Efimov, Perruquetti, & Richard, 2013; Richard, 2003; Yakoubi & Chitour, 2007; Ye, 2011; Zhang, Liu, & Zhang, 2013; Zhou, 2014; Zhou, Duan, & Lin, 2010; Zhou, Li, Zheng, & Duan, 2012). In this paper, we consider a chain of integrators with unknown time-varying delays in both states and input as

$$\begin{aligned} \dot{x}_i &= x_{i+1}(t-\tau_{i+1}(t)), \quad i = 1, \ldots, n-1, \\ \dot{x}_n &= u(t-\tau_{n+1}(t)) \\ y &= x_1(t-\tau_1(t)) \end{aligned} \tag{1}$$

where $x = [x_1, \ldots, x_n]^T \in R^n$ is the state, $u \in R$ is the input, $y \in R$ is the output of the system. Also, $x_i(t-\tau_i(t))$ and $u(t-\tau_{n+1}(t))$ denote each corresponding delayed state and input from x_i, $i = 1, \ldots, n$, and u, respectively. Regarding the delays in (1), we assume the following condition.

Assumption 1. There exist unknown $\bar{\tau}_i$ such that $0 \le \tau_i(t) \le \bar{\tau}_i$, $i = 1, \ldots, n+1$ for all $t \ge 0$.

Following Assumption 1, the initial conditions are given as $x_i(t+\theta)|_{t=0} = \nu_i(\theta)$, $-\bar{\tau}_i \le \theta \le 0$ for $i = 1, \ldots, n$ and $u(t+\theta)|_{t=0} = \nu_{n+1}(\theta)$, $-\bar{\tau}_{n+1} \le \theta \le 0$. Here, we formally address our control problem as follows:

Problem statement: To globally asymptotically regulate the system (1) under Assumption 1 by an output feedback controller.

Note that the system (1) has some challenging issues. There are unknown time-varying delays in both state and input. Assumption 1 indicates that all time-varying delays of (1) can be both arbitrarily large and fast-varying and there is no information on their bounds sizes and time-varying rates. Moreover, the system output y is actually different from the system state x_1 due to the unknown delay $\tau_1(t)$. In many existing results, it has been often assumed as $y = h(x)$, not like $y = h(x(t-\tau(t)))$. The novelty of our considered problem with respect to the existing results is stated in the following.

There have been a number of results that have specifically studied the control problems of a chain of integrators with delays (Choi & Lim, 2006, 2010; Karafyllis, 2006; Mazenc, Mondié, & Niculescu, 2003; Zhou et al., 2012). When $\tau_1(t) = \cdots = \tau_n(t) = 0$, $\tau_{n+1}(t) = \tau$, Assumption 1 reduces to the conditions in Choi and Lim (2006),

☆ This work was supported by the Dong-A University research fund. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Hiroshi Ito under the direction of Editor Andrew R. Teel.

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http://dx.doi.org/10.1016/j.automatica.2015.05.019
0005-1098/

Mazenc et al. (2003) where a switching output feedback controller, a state feedback controller, and a static output feedback controller are proposed, respectively. When $\tau_1(t) = \cdots = \tau_n(t) = 0$, $\tau_{n+1}(t) = \tau(t)$, Assumption 1 reduces to the condition in Choi and Lim (2010) where an adaptive output feedback controller is developed. When $\tau_i(t) = \tau_{i-1}$, $i = 2, \ldots, n+1$ and there is no output, Assumption 1 reduces to the conditions in Karafyllis (2006); Zhou et al. (2012) where a state feedback controller is developed for finite-time stabilization in Karafyllis (2006) and a low-gain state feedback controller is developed in Zhou et al. (2012). Thus, from a control problem viewpoint, our considered problem is a generalized one over the aforementioned results.

A chain of integrators system can be viewed as a special case of feedforward systems or lower triangular systems and there have been several results on the control of feedforward systems with delays (Koo et al., 2012; Ye, 2011), and lower triangular systems with delays (Bekiaris-Liberis & Krstic, 2010; Zhang et al., 2013). These results have their own merits in achieving robust global stabilization, dealing with uncertain delays and high-order terms. However, the results of Bekiaris-Liberis and Krstic (2010), Koo et al. (2012), Ye (2011) commonly deal with constant delays. In Zhang et al. (2013), the main integrators and the control input do not have delays and time-varying delays only exist in the nonlinearity part. For these obvious reasons, the results of Bekiaris-Liberis and Krstic (2010); Koo et al. (2012); Ye (2011); Zhang et al. (2013) are not applicable to our considered system.

There have been also results on general linear time-delay systems. In Krstic (2010), the author provides a new predictor-based controller to delay with uncertain delays in the input, but the delay is not unknown like our Assumption 1 and delay only exists in the input. In Bresch-Pietri et al. (2012), they mainly design a backstepping-based controller in order to deal with constant input delay and disturbance in the linear system. In Jankovic (2010), the author provides finite spectrum assignment and recursive predictor designs for linear delay systems that have constant delays in the input. The results of Lin and Fang (2007) commonly share the constant input delays cases for a class of linear systems. The saturation type controller for linear systems subject to input with a constant delay is demonstrated in Yakoubi and Chitour (2007) and a finite dimensional time varying controller based on the truncated predictor feedback is studied in Zhou (2014). In Polyakov et al. (2013), they have commonly considered a linear system that has unknown time-varying input delay, but there is no delay in the state. For nonlinear time-delay systems, Bekiaris-Liberis et al. (2012) construct regional stability results in order to compensate the state dependent delays in the states. In Bekiaris-Liberis and Krstic (2013), the authors consider the state dependent input delayed systems with no limitations for the delay size at the origin.

Overall, all the aforementioned observations with the existing results clearly show that their results are not applicable to our considered system. To our best knowledge, as of now, our control problem has not been solved before. In the next section, we introduce our newly designed adaptive controller along with new system analysis techniques that involve a new transformation.

2. Main result

We propose an observer based output feedback controller with a dynamic gain as

$$u = K(\gamma(t))z \tag{2}$$

$$\dot{z} = Az + Bu - L(\gamma(t))(y - Cz) \tag{3}$$

where $K(\gamma(t)) = [\frac{k_1}{\gamma(t)^n}, \ldots, \frac{k_n}{\gamma(t)}]$, and $L(\gamma(t)) = [\frac{l_1}{\gamma(t)}, \ldots, \frac{l_n}{\gamma(t)^n}]^T$, and the matrices (A, B) are Brunovsky canonical pair ($A = [e_{ij}]$, $i = 1, \ldots, n$, $j = 1, \ldots, n$ with $e_{ij} = 1$ if $j = i+1$ and $e_{ij} = 0$ if $j \neq i+1$ for $i = 1, \ldots, n$, and $B = [0, \ldots, 0, 1]^T$). The dynamic gain $\gamma(t)$ will be given later.

Here, we address some mathematical setups and notations.

- Setup: Let $A_K(\gamma(t)) = A + BK(\gamma(t))$, $A_L(\gamma(t)) = A + L(\gamma(t))C$, $K = K(1)$, $L = L(1)$, $A_j(1) = A_j$, $j = K, L$. We define $E_{\gamma(t)} = \text{diag}[1/\gamma(t)^{n-1}, \ldots, 1/\gamma(t), 1]$. Then, if given that A_K and A_L are Hurwitz, from Choi and Lim (2006), we can deduce Lyapunov equations of $A_j^T(\gamma(t))P_j(\gamma(t)) + P_j(\gamma(t))A_j(\gamma(t)) = -\gamma(t)^{-1}E^2_{\gamma(t)}$ with $P_j(\gamma(t)) = E_{\gamma(t)}P_jE_{\gamma(t)}$ and $A_j^TP_j + P_jA_j = -I$ for $j = K, L$ where I denotes an $n \times n$ identity matrix. From Lei and Lin (2007), it is clear that there exist positive constants π_{j_1} and $\pi_{j_2} > 0$ such that $\pi_{j_1}I \leq P_jD + DP_j \leq \pi_{j_2}I$, $j = K, L$, where $D = \text{diag}[\frac{2n-1}{2}, \ldots, \frac{2(n-i)+1}{2}, \ldots, \frac{1}{2}]$, $i = 1, \ldots, n$.
- Notation: We follow the convention that $\zeta_{n+1} = x_{n+1} = u$ and $\sum_{j=r_1}^{r_2} f_j = 0$ if $r_2 < r_1$. Also, we let $f(t, x)|_{x=\tilde{x}}$ be denoted that x in $f(t, x)$ is replaced by $\tilde{x}$. We denote $\lambda_{\max}(\bar{M})$ and $\lambda_{\min}(\bar{M})$ are the maximum and minimum eigenvalues of the matrix $\bar{M}$, respectively. The norms $\|x\|$ and $\|x\|_1$ denote Euclidean norm and 1-norm, respectively.

Next, consider the dynamic gain as

$$\dot{\gamma}(t) = \frac{e^{\sqrt{2t}/(4M)}(|y| + \|z\|_1)}{1 + e^{\sqrt{2t}/(4M)}(|y| + \|z\|_1)}\gamma(t)^{-n-1} \tag{4}$$

with $M = \max\{\lambda_{\max}(P_K), \lambda_{\max}(P_L)\}$ and $\gamma(\theta) = 1$ for $\theta \leq 0$.

Remark 1. We note two things: (i) even though our considered system (1) contains several time-varying delays, our designed controller (2)–(4) does not need any of delay information at all for our control purpose. That is, we will show that our controller with Luenberger-like observer with specially designed dynamic-gain fulfills our control goal; (ii) The delays in system (1) do not need to be slowly varying, or similar to each other at all. The only clear requirement is that they are bounded.

Now, we show the main theorem.

Theorem 1. *Under Assumption 1, select K and L such that A_K and A_L are Hurwitz and obtain P_j of $A_j^TP_j + P_jA_j^T = -I$, $j = K, L$. Then, the output feedback controller (2)–(4) globally regulates the system (1). Also, the dynamic gain $\gamma(t)$ converges to a finite constant as $t \to \infty$.*

Proof. Consider the transformation $\zeta = T(x)$ as follows.

$$\begin{aligned}\zeta_i &= x_i + \int_{t-\bar{\tau}_{i+1}}^{t} x_{i+1}(s)ds, \quad i = 1, \ldots, n-1 \\ \zeta_n &= x_n + \int_{t-\bar{\tau}_{n+1}}^{t} u(s)ds.\end{aligned} \tag{5}$$

Note the following relation as

$$x_i(t - \tau_i(t)) - x_i(t - \bar{\tau}_i) = \int_{t-\bar{\tau}_i}^{t-\tau_i(t)} \dot{x}_i(s)ds, \quad i = 1, \ldots, n, \tag{6}$$

$$u(t - \tau_{n+1}(t)) - u(t - \bar{\tau}_{n+1}) = \int_{t-\bar{\tau}_{n+1}}^{t-\tau_{n+1}(t)} \dot{u}(s)ds. \tag{7}$$

From (5) to (7), and $\dot{u} = \dot{K}(\gamma(t))z + K(\gamma(t))\dot{z}$, the system (1) is transformed into

$$\begin{aligned}\dot{\zeta}_i &= \zeta_{i+1} + \delta_i(t, \zeta, u), \quad i = 1, \ldots, n-1 \\ \dot{\zeta}_n &= u + \delta_n(t, \zeta, n)\end{aligned} \tag{8}$$