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A unified model approach for probability response analysis of structure-acoustic system with random and epistemic uncertainties

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ABSTRACT

The uncertainties of structure-acoustic system can be generally characterized as the random uncertainty and the epistemic uncertainty. Traditional methods for response analysis of structure-acoustic system with hybrid random and epistemic uncertainties are developed by integrating different mathematical theories, such as the probability theory and the interval analysis. In this paper, the *Evidence-Theory-based Unified Uncertain Model* (ETUUM) is introduced to deal with hybrid random and epistemic uncertain structure-acoustic problem. In ETUUM, both epistemic and random uncertainties are modeled by using evidence theory. For uncertainty quantification of ETUUM of structure-acoustic system with hybrid random and epistemic uncertainties, a new hybrid uncertainty analysis method named as the *Evidence-Theory-based Arbitrary Polynomial Chaos Method* (ETAPCM) is proposed. In ETAPCM, the response of ETUUM in the range of variation of the uncertain parameter is approximated by the *arbitrary Polynomial Chaos* (aPC) expansion, through which the uncertainty property of the response can be efficiently obtained. As the aPC expansion can provide a free choice of the polynomial basis, the optimal polynomial basis of polynomial chaos expansion for the epistemic uncertain parameter and the random parameter with arbitrary probability distribution can be obtained by using aPC expansion. The effectiveness of the proposed methodology has been investigated by comparing it with the conventional polynomial chaos and evidence theory based hybrid uncertainty analysis method as well as the probability theory based hybrid uncertainty analysis method.

1. Introduction

Uncertainties related to the manufactory error, unpredictable environment and other factors are widely involved in engineering. Without considering these uncertainties, the results obtained via deterministic analysis methods may be unfeasible. Normally, uncertainties can be categorized as two distinct groups [1]: random uncertainty and epistemic uncertainty. Random uncertainty, also called as aleatory uncertainty, can be characterized by the well defined probability distribution and is always modeled by using the probability theory [1,2]. Conversely, the epistemic uncertainty arises from the lack of knowl-

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edge of probabilistic information. For the epistemic uncertainty, probability representation is inappropriate since some assumptions should be made for the *probability density function* (PDF) of uncertain parameters. To deal with epistemic uncertainties, lots of non-probabilistic mathematical theories have been developed, such as the interval analysis [3,4], the fuzzy sets [5,6], the p-box theory [7,8] and the evidence theory [9,10].

In the previous study, most of the uncertainty analysis methods are focused on either random problem or epistemic uncertain problem. However, in practical engineering, the random uncertainty and the epistemic uncertainty may exist simultaneously. Thus, alternative mathematical model should be developed to deal with the hybrid random and epistemic uncertainties. The most straightforward technique for uncertain problem with random and epistemic uncertainties is the hybrid mathematical model, such as the hybrid random and interval model [11–20], the hybrid random and fuzzy model [21,22]. In comparison to the uncertainty analysis of pure probability model or pure non-probability model, the quantification of hybrid mathematical model may be more challenging. The main difficulty by using the hybrid mathematical model is that two distinct mathematical frameworks should be properly integrated in the process of uncertainty propagation, which can make the procedure of hybrid uncertainty analysis rather complicated. To overcome the computational complexity caused by the use of hybrid mathematical model, the unified mathematical model has been recently developed for structural analysis with mixed uncertainties by using evidence theory [23]. The evidence theory has the ability to handle the probability parameters, the non-imprecise probability parameters and the interval parameters. Thus, both random and epistemic uncertainties can be modeled by using evidence theory in a unified form. Besides, the evidence theory can deal with conflict uncertainty information from different experts. Due to these above advantages, this paper will focus on the application of evidence theory for hybrid uncertainty analysis.

Evidence theory starts from the *Basic Probability Assignments* (BPA) on the input variables. Under evidence theory, the evidence variable is represented by many subintervals that are called as focal elements. As one need to calculate the bounds of the system response over each focal element, onerous computational cost is inevitable in uncertainty quantification under evidence theory. In previous years, extensive researches have been dedicated to reduce the excessive computational cost associated with repetitively response interval analysis. Bea et al employs the first- and second-order perturbation method to improve the efficiency of interval analysis over each focal element [24,25]. To improve the accuracy of evidence-theory-based first-order perturbation method, Yin et al. introduced the sub-interval perturbation method for mid-frequency analysis of built-up system with evidence variable [26]. The main disadvantage of the evidence-theory-based perturbation method is that the computational cost will increase exponentially with the number of the focal elements. However, the evidence variable may include a large number of focal elements, especially when the evidence theory is used for random uncertainty analysis [23]. To solve uncertain problems with large number of focal elements, the response surface method has been proposed for evidence-theory-based uncertainty analysis [27]. By using the response surface method, the response in the range of variation of evidence variable is approximated by a simple function, through which the response interval analysis can be efficiently obtained. Recently, the polynomial chaos method has also been developed to construct the approximation function for the response of uncertain system with evidence variables [23,28]. Both the polynomial chaos method and the response surface method can significantly reduce the computational cost of evidence-theory-based uncertainty analysis. But the efficiency of the polynomial chaos method for uncertainty analysis is generally higher than that of the response surface method [29]. Thus, the polynomial chaos method has gained an increasing interest for evidence-theory-based uncertainty analysis.

As we mentioned previously, many methods have been developed for uncertainty analysis under evidence theory. Especially, the polynomial chaos method has shown its good accuracy and efficiency for evidence-theory-based uncertainty analysis. However, in the application of polynomial chaos method for evidence-theory-based uncertainty analysis of hybrid random and epistemic uncertain problem, there are still some important issues remaining unsolved. Firstly, only the Legendre polynomial is used to construct the polynomial chaos expansion for the approximation of response of hybrid random and epistemic uncertain problem related to the epistemic variable, while the optimal choice of polynomial basis of the polynomial chaos expansion for the epistemic uncertainties has not been addressed [23]. Secondly, the polynomial chaos expansion for the response of hybrid random and epistemic uncertain problem related to the random variable is established by using the *generalized Polynomial Chaos* (gPC), whereas the accuracy and efficiency of the gPC based method may be deteriorated for uncertain problem with the probability distribution out of Askey scheme. This is because the optimal polynomial basis of polynomial chaos expansion for uncertainty analysis with the probability distribution out of Askey scheme cannot be obtained by using gPC [30,31]. Thirdly, a unified mathematical model for hybrid random and epistemic uncertain structure-acoustic problem is still unreported. The response analysis of hybrid random and epistemic uncertain structure-acoustic system plays an important role in the vibration and noise control of engineering products. In the last decades, there is an increasing interest for solving the hybrid random and epistemic uncertain structure-acoustic problem. Up to now, the uncertainty analysis methods for hybrid random and epistemic uncertain structure-acoustic problem are generally developed based on the hybrid probability and interval model [32–36]. However, the response analysis of the hybrid probability and interval model of structure-acoustic system is not inconvenient to be implemented. This is because the uncertainty responses related to the random and interval variables should be properly integrated by using the hybrid probability and interval model. Therefore, it is desirable to develop a unified mathematical model which can effectively handle both random and epistemic uncertainties of the structure-acoustic problem.

This paper aims to develop a polynomial chaos-based unified model approach for response analysis of structure-acoustic problem with hybrid random and epistemic uncertainties. In order to model the random and epistemic uncertainties of