Mechanical Systems and Signal Processing 111 (2018) 628–642

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Stabilization in finite time for fractional-order hyperchaotic electromechanical gyrostat systems ☆

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ARTICLE INFO

Article history:
Received 18 October 2017
Received in revised form 1 March 2018
Accepted 6 April 2018

Keywords:
Fractional-order gyrostat system
Hyperchaos
Stabilization in finite time
Adaptive control law
Unknown parameter
Robustness

ABSTRACT

This paper is concerned with adaptive robust stabilization in finite time for fractional-order chaotic and hyperchaotic Electromechanical Gyrostat Systems (EGSs) with unknown parameters and uncertainties. Firstly, a fractional-order EGS mathematical model is proposed, and the existence of chaotic and hyperchaotic attractors is verified with the related Lyapunov exponents, phase portraits and Poincaré sections. Then, fractional-order adaptive laws are proposed to estimate the unknown parameters. Based on the fractional-order convergence theory in finite time, a novel discontinuous state feedback controller with the proposed fractional-order adaptive laws is designed to stabilize the fractional-order EGSs in finite time globally, and the stabilization conditions are analytically addressed. Finally, numerical simulations are employed to demonstrate the validity of the theoretical results.

1. Introduction

Fractional-order calculus disposes of the generalization of differentiation and integration up to an arbitrary order [1–5], and has attracted increasing attention in physics and engineering over the past several decades [6,7]. On one hand, it was found that fractional-order calculus is a useful instrument for modeling many systems better and describing complex physical phenomena more elegantly, especially for systems of memory and hereditary properties, such as mechanical system [8], dynamic backlash [5], various materials and processes [9], fluid mechanics [10], bioengineering [11], wind turbine generators [12], chemical system [13], and oscillation of earthquakes [14]. On the other hand, it is convenient to design robust controllers with the utilization of fractional-order calculus [15]. Hence, there were many contributions introducing fractional-order theory to pre-existing integer-order mathematical models, see [16–19] and references therein.

The existence of chaotic and hyperchaotic attractors can be verified with some methods. Positive Lyapunov exponents indicate chaos for dynamical systems [20]. In 1979, Rössler has been reported firstly hyperchaos, which is defined as a high-dimensional chaotic attractor with more than one positive Lyapunov exponents [21]. Moreover, there are many means that confirm the existence of chaotic and hyperchaotic attractors for a dynamical systems, such as the related phase portraits and Poincaré sections. It is worth mentioning that the complex dynamical behaviors of dynamical systems can be effectively investigated by means of numerical simulations [22–25].

☆ This work was supported by the Natural Science Foundation of Hebei Province of China (A2018203288) and High level talent support project of Hebei province of China (C2015003054).

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https://doi.org/10.1016/j.ymssp.2018.04.009
0888-3270/

Chaos control has attracted increasing interest of many investigators, since the ground breaking work of Ott et al. [26]. After that, various control schemes have been developed to stabilize chaotic systems, such as backstepping control [27], sliding mode control [28], passive control [29], impulsive control [30] and fuzzy control [31]. Analogous results for fractional-order chaotic and hyperchaotic systems can be seen in [32–35].

The EGSs can be described by four-variables non-autonomous time-varying ordinary differential equations, and exhibit very complex dynamical behaviors, for instance, chaos. There were many literatures on EGSs, since traditional mechanical systems can be steered and used easier with the help of electromechanical systems [36]. In [36,37], the dynamic behaviors of chaotic EGSs have been investigated extensively. Meanwhile, chaos control, chaos anticontrol and chaos synchronization in the EGS have been discussed. The global synchronization of non-autonomous chaotic EGSs via variable substitution control has been explored in [38]. The sufficient criteria for global synchronization of a class of third-order non-autonomous chaotic EGSs has been reported in [39], via linear state error feedback control.

It should be pointed out that, the above-mentioned work is to stabilize or synchronize EGSs asymptotically, namely, the complete synchronization or synchronization cannot be accomplished in finite time. Whereas, the maximum value of stabilization or synchronization time requires to be explicitly evaluated in practical engineering applications. In [40], based on adaptive control technology, the finite-time stabilization and synchronization of non-autonomous EGSs have been studied.

It should be noted that, the all work mentioned above is based on the model described by integer-order ordinary differential equations for EGSs, and to the best of our knowledge, there is no effort being made in the literature to investigate the fractional-order EGSs so far. However, fractional-order calculus can allow the model to take into account peculiarities that the classical integer-order model is unable to capture [5,41–48], more importantly, a large volume of applications can benefit from embedding novel fractional-order concepts into refined models. Therefore, the fractional-order mathematical model with respect to EGSs is interesting in theory and application.

Motivated by the previous discussion, in this paper, our aim is to propose a fractional-order EGS mathematical model and discuss the robust stabilization in finite time for the proposed fractional-order EGSs with unknown parameters and uncertainties. Although chaos and hyperchaos, by now, are known to present in virtually any system of equations of the type considered in this paper, the verification for existence of the chaotic and hyperchaotic attractors in the proposed fractional-order EGSs is provided to be sure, via the Lyapunov exponents, phase portraits and Poincaré sections. Based on fractional Lyapunov function strategy, the robust stabilization conditions are presented.

The crucial novelty of our contribution lies in following aspects:

1. The fractional-order dynamic model with respect to EGSs is developed by fractional-order differential equation systems.
2. The existence of chaotic and hyperchaotic attractors in the proposed fractional-order EGSs are verified.
3. The novel robust controller, which includes discontinuous factors, is designed to realize the global stabilization goal in finite time.
4. The unknown parameters are estimated via new fractional-order adaptive laws proposed.
5. The conditions and the upper bounds of the settling time for the global robust stabilization in finite time are addressed and evaluated, respectively.

The rest of this paper is organized as follows. In Section 2, some definitions of fractional-order calculus and relevant properties are presented, respectively. And then the fractional-order EGSs model is developed by introducing Caputo fractional-order differential operator. In Section 3, the existence of the chaotic and hyperchaotic attractors in the proposed fractional-order EGSs are verified. Then the fractional-order adaptive law and controller are designed, and the global robust stabilization in finite time is analytically discussed. In Section 4, numerical simulations are provided. Some conclusions are drawn in Section 5.

Notation: R denotes the set of real numbers, R^n denotes the n-dimensional Euclidean space, $R^{m\times n}$ denotes the set of all $m \times n$ real matrices. Given the vector $w = [w_1, w_2, \ldots, w_n]^T \in R^n$, w^T denotes the transpose of w, and $\|w\| = \left(\sum_{i=1}^{n} w_i^2\right)^{\frac{1}{2}}$ denotes the Euclidean norm of w in R^n. If function $V : R^n \to R$ be: (i) regular in R^n; (ii) positive definite, i.e., $V(x) > 0$ for $x \neq 0$, and $V(0) = 0$; (iii) radially unbounded, i.e., $V(x) \to +\infty$ as $\|x\| \to +\infty$, then $V(\cdot)$ is said to be C-regular. $\mathrm{sgn}(\cdot)$ denotes the signum function.

2. Preliminaries and model description

2.1. Fractional-order integral and derivative

In this subsection, some basic definitions about fractional-order calculus are recalled, and some useful lemmas are presented.

Definition 2.1. ([3]) The Riemann-Liouville fractional integral of order α for a function $f(t) : [0, +\infty) \to R$ is defined as

$$ {}_0^R I_\alpha^t f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau, $$

where $\alpha > 0$.