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Simultaneous identification of bi-directional paths in closed-loop systems with coloured noise[☆]

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ABSTRACT

In this paper, a novel instrumental variable (IV) based identification method is proposed for closed-loop systems in the presence of coloured noise. The key technique lies in constructing an interleaved information matrix with respect to a multiple model structure formulated for the bi-directional paths. Then by utilizing UD factorization, all the parameter estimates for both forward and backward path models with orders possibly from zero to n, as well as the corresponding minimum loss function values, can be obtained simultaneously. Simulation results are provided to show the effectiveness of the proposed method.

1. Introduction

Parameter estimation approaches based on least-squares principle are widely used in many fields including linear prediction problems and digital signal processing. They are often summed up in the problem of optimizing a set of overdetermined equations

$$\mathbf{y} = \mathbf{X}\boldsymbol{\theta} + \mathbf{e}, \tag{1}$$

with the objective $J = \min_{\boldsymbol{\theta}} \|\mathbf{y} - \mathbf{X}\boldsymbol{\theta}\|^2$, where $\mathbf{y} \in R^{m\times 1}$ represents the observation vector, $\mathbf{X} \in R^{m\times n}$ is a data matrix, $\boldsymbol{\theta} \in R^{n\times 1}$ denotes a coefficient vector to be estimated, and $\mathbf{e} \in R^{m\times 1}$ denotes the error vector.

However, owing to its poor numerical performance, least-squares problem may not be well solved by $\widehat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \|\mathbf{y} - \mathbf{X}\boldsymbol{\theta}\|^2 = \left[\mathbf{X}^{\mathrm{T}}\mathbf{X}\right]^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{y}$ in practice, especially when the matrix $\mathbf{X}$ is ill conditioned (Golub & van Loan, 1989). Thus some alternative parameter estimation methods by adopting matrix factorization techniques are often utilized (Golub & van Loan, 1989). In Niu, Ljung and Björck (1996), matrix decomposition techniques were investigated by incorporating a multiple model least-squares (MMLS) structure, with which n sets of linear equations can be solved without any extra computational cost. In addition, the MMLS approach was found to attain improved numerical performance by appropriately choosing the matrix factorization technique. Inspired by this, Niu et al. presented a decomposition-assisted least-squares method, which is known as augmented upper diagonal identification (AUDI) algorithm, for open loop systems (Niu & Fisher, 1995; Niu, Fisher, Ljung, & Shah, 1994). In contrast to traditional least-squares algorithms, the AUDI algorithm has the following advantages: (i) More information involved in the data matrix, i.e. the augmented information matrix (AIM), can be extracted; (ii) The extracted information has clear physical significance; (iii) By performing UD factorization on the AIM, parameter estimates of the systems with orders from zero to n as well as the corresponding minimum loss function values can be obtained simultaneously from the resulting matrices $\mathbf{U}$ (the parameter matrix) and $\mathbf{D}$ (the loss function matrix).

Note that in the AUDI algorithm, an open issue remains that is the physical significance of the even columns of parameter matrix and loss function matrix is left unclear for open-loop identification. In Jiang, Yang, Wang, and Huang (2015), the authors investigated the problem of simultaneous identification of bi-directional path models in a closed-loop system in the presence of white noise. An interleaved data pair upper diagonal (IDPUD) algorithm was proposed in which both the odd and even columns of parameter matrix and loss function matrix were utilized.

In this paper, we further extend the method in Jiang et al. (2015) to solve the closed-loop identification problem for systems with coloured noise, which commonly exist in industrial processes. A new IDPUD identification method is proposed, in which the

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instrumental variable (IV) technique (Söderström & Stoica, 1983) and the MMLS approach are incorporated.

2. Problem formulation

We consider a class of closed-loop processes with two data collection ends. The forward and backward paths with x and y being the corresponding outputs are modelled by the following linear time-invariant discrete-time systems

$$\begin{cases} x(k) = \dfrac{B(z)}{A(z)} y(k) + e_x(k), & \text{(a)} \\ y(k) = \dfrac{Q(z)}{P(z)} x(k) + r(k) + e_y(k), & \text{(b)} \end{cases} \tag{2}$$

where

$$\begin{cases} A(z) = 1 + a_1 z^{-1} + \cdots + a_{n_x} z^{-n_x}, \\ B(z) = b_d z^{-d} + \cdots + b_{n_x} z^{-n_x}, \\ P(z) = 1 + p_1 z^{-1} + \cdots + p_{n_y} z^{-n_y}, \\ Q(z) = q_c z^{-c} + \cdots + q_{n_y} z^{-n_y}. \end{cases} \tag{3}$$

In the above expressions, z is the forward shift operator. Eqs. (2)(a) and (2)(b) represent the models of the forward and backward paths, respectively. Integers n_x and n_y denote the corresponding orders, and a_j, b_j, p_j and q_j are the model parameters. Nonnegative integers c and d are the delays in the backward and forward paths, respectively. Variables e_x and e_y denote coloured noise sequences. Reference signal r describes a known external signal assumed to be uncorrelated with e_x and e_y.

The problem of simultaneous identification of both forward and backward paths of the closed-loop process with coloured noise e_x and e_y in (2)(a)–(2)(b) is investigated in this paper.

The following assumptions are imposed.

Assumption 1. A delay exists in at least one path in the closed-loop process, i.e., $c + d > 0$ or $\lim_{z\to\infty} \frac{B(z)}{A(z)} \cdot \lim_{z\to\infty} \frac{Q(z)}{P(z)} = 0$.

Remark 1. Assumption 1 is required in this paper for two reasons. (i) The systems with Assumption 1 satisfied can be frequently encountered in practice (Anderson & Gevers, 1982; Sin & Goodwin, 1980). For example, in computer control systems, the sample and hold characteristics of digital controllers will inevitability result in delays. (ii) Assumption 1 is needed to maintain the upper triangle form of parameter matrix in the multiple model structure formulated for simultaneous identification of both forward and backward paths. More details will be given in the subsequent section.

Assumption 2 (*Anderson & Gevers, 1982*)**.** *Process noise sequences e_x and e_y are uncorrelated, i.e., $e_x(i) \perp e_y(j)$, for any i and j.*

3. Interleaved data pair upper diagonal identification algorithm based on instrumental variables technique

In this section, the IDPUD identification algorithm in Jiang et al. (2015) will be extended by incorporating the instrumental variables technique. An interleaved form of multiple model structure corresponding to the forward and backward path models with orders from 0 to a sufficiently large n will be formulated. Then, an interleaved information matrix (IIM) will be elaborately constructed. It will be shown that all the parameter estimates for both forward and backward path models as well as the corresponding loss function values can be obtained by performing UD factorization on the IIM.

3.1. An interleaved form of multiple model structure

Without loss of generality, suppose that $d > 0$, whereas $c = 0$. From (2)(a) and (3), if the system order of the forward path model is i ($i = 0, 1, 2, \ldots, n$), then (2)(a) can be rewritten as

$$\begin{aligned} x(k) = {} & e_x(k) - \theta_1^{(i)} x(k-i) + \theta_2^{(i)} y(k-i) \\ & - \theta_3^{(i)} x(k-i+1) + \theta_4^{(i)} y(k-i+1) \\ & + \cdots - \theta_{2i-1}^{(i)} x(k-1) + \theta_{2i}^{(i)} y(k-1). \end{aligned} \tag{4}$$

Obviously, odd parameters $\theta_1^{(i)}, \theta_3^{(i)}, \ldots, \theta_{2i-1}^{(i)}$ correspond to $a_i, a_{i-1}, \ldots, a_1$, respectively; even parameters $\theta_2^{(i)}, \theta_4^{(i)}, \ldots, \theta_{2(i+1-d)}^{(i)}$ correspond to $b_i, b_{i-1}, \ldots, b_d$, respectively; and $\theta_{2(i+2-d)}^{(i)}, \ldots, \theta_{2i}^{(i)}$ are all zero.

We use $\widehat{\theta}_l^{(i)}$, $l = 1, 2, \ldots, 2i$, to denote the estimates of $\theta_l^{(i)}$, which will be defined explicitly in Section 3.2. Define

$$\begin{aligned} \widehat{x}^{(i)}(k) = {} & -\widehat{\theta}_1^{(i)} x(k-i) + \widehat{\theta}_2^{(i)} y(k-i) - \widehat{\theta}_3^{(i)} x(k-i+1) \\ & + \widehat{\theta}_4^{(i)} y(k-i+1) + \cdots - \widehat{\theta}_{2i-1}^{(i)} x(k-1) + \widehat{\theta}_{2i}^{(i)} y(k-1), \end{aligned} \tag{5}$$

then the residuals with respect to the forward path are introduced as

$$\widehat{e}_x^{(i)}(k) = x(k) - \widehat{x}^{(i)}(k). \tag{6}$$

Similarly, the residuals with respect to the backward path with order i, for $i = 0, 1, \ldots, n$, are introduced as

$$\widehat{e}_y^{(i)}(k) = y(k) - \widehat{y}^{(i)}(k), \tag{7}$$

where

$$\begin{aligned} \widehat{y}^{(i)}(k) = {} & \widehat{\alpha}_1^{(i)} x(k-i) - \widehat{\alpha}_2^{(i)} y(k-i) \\ & + \widehat{\alpha}_3^{(i)} x(k-i+1) - \widehat{\alpha}_4^{(i)} y(k-i+1) \\ & + \cdots + \widehat{\alpha}_{2i-1}^{(i)} x(k-1) - \widehat{\alpha}_{2i}^{(i)} y(k-1) + \widehat{\alpha}_{2i+1}^{(i)} x(k), \end{aligned} \tag{8}$$

with $\widehat{\alpha}_l^{(i)}$, $l = 1, 2, \ldots, 2i+1$, representing the estimates of the parameters in $P(z)$ and $Q(z)$ in (3).

Define an interleaved data vector as

$$\boldsymbol{\varphi}(k) = [-x(k-n), y(k-n), \ldots, -x(k), y(k)]^{\mathrm{T}}, \tag{9}$$

with n chosen sufficiently large, i.e.

$$n \geq \max\left\{n_x, n_y\right\}. \tag{10}$$

By combining the above $2n+2$ models derived for both forward and backward paths, a multiple model structure can be formulated as

$$\boldsymbol{\varphi}^{\mathrm{T}}(k)\mathcal{U} = \mathcal{E}^{\mathrm{T}}(k), \tag{11}$$

where

$$\mathcal{U} = \begin{bmatrix} 1 & \widehat{\alpha}_1^{(0)} & \widehat{\theta}_1^{(1)} & \widehat{\alpha}_1^{(1)} & \cdots & \widehat{\alpha}_1^{(n-1)} & \widehat{\theta}_1^{(n)} & \widehat{\alpha}_1^{(n)} \\ & 1 & \widehat{\theta}_2^{(1)} & \widehat{\alpha}_2^{(1)} & \cdots & \widehat{\alpha}_2^{(n-1)} & \widehat{\theta}_2^{(n)} & \widehat{\alpha}_2^{(n)} \\ & & 1 & \widehat{\alpha}_3^{(1)} & \cdots & \widehat{\alpha}_3^{(n-1)} & \widehat{\theta}_3^{(n)} & \widehat{\alpha}_3^{(n)} \\ & & & 1 & \cdots & \widehat{\alpha}_4^{(n-1)} & \widehat{\theta}_4^{(n)} & \widehat{\alpha}_4^{(n)} \\ & & & & 1 & \vdots & \vdots & \vdots \\ & & & & & 1 & \widehat{\theta}_{2n}^{(n)} & \widehat{\alpha}_{2n}^{(n)} \\ & & & & & & 1 & \widehat{\alpha}_{2n+1}^{(n)} \\ & & & & & & & 1 \end{bmatrix}, \tag{12}$$

$$\mathcal{E}(k) = [-\widehat{e}_x^{(0)}(k-n), \widehat{e}_y^{(0)}(k-n), \ldots, -\widehat{e}_x^{(n)}(k), \widehat{e}_y^{(n)}(k)]^{\mathrm{T}}. \tag{13}$$