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Stabilization of continuous-time linear systems subject to input quantization[☆]

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ABSTRACT

This paper deals with the stabilization of continuous-time linear time-invariant systems subject to uniform input quantization. Specifically, the right-hand side of the closed-loop system is rewritten as a linear system subject to a discontinuous perturbation due to the quantization error. Then, the controller design is performed to achieve finite-time convergence of the closed-loop trajectories toward a compact invariant set surrounding the origin. Furthermore, a computationally tractable design procedure for the proposed controller based on linear matrix inequalities, and some insights on the simulation of the closed-loop system are presented. In addition, the effectiveness of the proposed control design procedure is shown in a numerical example.

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1. Introduction

Recent technology enhancement has enabled the control of dynamical systems via digital controllers. When a continuous-time plant is controlled through a digital controller, side effects such as time-delays, asynchronism, quantization, (see Elia and Mitter (2001) and references therein), can turn into an excessive performance degradation like the appearance of limit cycles, chaotic phenomena or even instability of the closed-loop system, (Ceragioli & De Persis, 2007; Delchamps, 1990). Concerning the effect of quantization in control systems, such a topic has been extensively addressed by researchers over the last years; see, *e.g.*, Brockett and Liberzon (2000), Ceragioli and De Persis (2007), Coutinho, Fu, and de Souza (2010), Delchamps (1990), Fu and Xie (2005), Liberzon (2003), Sur and Paden (1997) and Tarbouriech and Gouaisbaut (2012) just to cite a few.

This paper pertains to the stabilization of continuous-time linear time-invariant plants with uniformly-quantized input via static state feedback control. Specifically, pursuing the general approach introduced in Delchamps (1990), we model the uniform quantizer as a discontinuous static isolated nonlinearity entering into the dynamics of the closed-loop system. At this stage, since the resulting closed-loop system is described by a discontinuous right-hand side differential equation, the existence of solutions to the closed-loop system is not guaranteed; see Filippov (1988). Therefore, to tackle the problem under consideration, we adopt, for the closed-loop system, the notion of solution due to Krasovskii; see Cortés (2008). Then, by the use of the sector conditions for the uniform quantizer presented in Ferrante, Gouaisbaut, and Tarbouriech (2014), coupled through S-procedure (see Boyd, El Ghaoui, Feron, and Balakrishnan (1997)) to a quadratic Lyapunov-like function, we propose a condition to guarantee the finite-time convergence of the closed-loop trajectories toward a compact invariant set surrounding the origin, (asymptotic stability is usually impossible to prove due to the deadzone effect induced by uniform quantization; see, *e.g.*, Ceragioli, De Persis, and Frasca (2011), Ferrante et al. (2014) and Tarbouriech and Gouaisbaut (2012)). Afterwards, via the use of the projection lemma (see Pipeleers, Demeulenaere, Swevers, and Vandenberghe (2009)), such a condition is turned into a design procedure based on the solutions of a convex optimization problem that in one shot

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provides: the controller gain, the invariant set wherein the closed-loop trajectories ultimately converge, while minimizing the size of such a set. Moreover, some insights on the simulation of the considered closed-loop system are discussed.

It is worthwhile to notice that, although the approach introduced in Delchamps (1990), (consisting in modeling quantizers as isolated nonlinearities), has enabled to build constructive design tools for quantized control systems, as for the case of other types of isolated nonlinearities such as, saturation, backlash etc., surprisingly no much work has been done in that direction, except for Coutinho et al. (2010), Fu and Xie (2005) and Tarbouriech and Gouaisbaut (2012), though the results in Coutinho et al. (2010) and Fu and Xie (2005) relate to discrete-time systems subjected to a logarithmic quantizer (deadzone-free). Specifically, the majority of the results available in the literature (see, *e.g.*, Ceragioli and De Persis (2007) and Liberzon (2003) and references therein), has dealt with the stability analysis of quantized closed-loop systems involving a controller designed while ignoring the presence of the quantizer. In this sense, this paper wants to fill this gap by proposing a design technique tailored to quantized systems. The contribution of this paper with respect to Tarbouriech and Gouaisbaut (2012) is twofold. On one hand, the novel sector conditions for the uniform quantizer presented in Ferrante et al. (2014) are for the first time exploited to design a static state feedback controller in the presence of uniform input quantization. On the other hand, the use of the projection lemma is considered to potentially improve the proposed design technique. Notice also that even if the spirit of the pursued approach is similar to Ceragioli and De Persis (2007) and Jayawardhana, Logemann, and Ryan (2011), the results presented in this paper allows to deal with multi-inputs systems, derive constructive conditions for the design of a state feedback controller and explicitly characterize the set wherein the closed-loop trajectories ultimately converge, directly from the knowledge of a Lyapunov-like function. Therefore, our results can be considered as complementary with respect to those in Ceragioli and De Persis (2007) and Jayawardhana et al. (2011).

The paper is organized as follows. Section 2 presents the system under consideration and the problem we solve. Section 3 is dedicated to the main results. Section 4 is devoted to numerical issues about the controller design. Moreover, some aspects on the simulation of the closed-loop system are discussed. Finally, Section 5 shows the effectiveness of the presented results in a numerical multi-inputs example.

Notation. The set $\mathcal{B}(x,\delta)$ denotes the δ radius closed Euclidean ball centered at x. $\mathbf{I}_n$ denotes the identity matrix and $\mathbf{0}$ denotes the null matrix (equivalently the null vector) of appropriate dimensions. For a matrix $A \in \mathbb{R}^{n\times m}$, A', $A_{(i)}$ denotes its ith row, and *trace*(A) denote its transpose and its trace, and $\mathrm{He}(A) = A + A'$. The matrix $\mathrm{diag}\{A_1, A_2, \ldots, A_n\}$ is the block-diagonal matrix having $A_1, A_2, \ldots, A_n$ as diagonal blocks and in symmetric matrices $\bullet$ stands for symmetric blocks. For a vector $x \in \mathbb{R}^n$, $x_{(i)}$ denotes its ith component, x' denotes its transpose, $|x|$ stands for the componentwise absolute value operator, $\mathrm{sign}(x)$ is the componentwise sign function, with $\mathrm{sign}(0) = 0$, and $\lfloor x \rfloor$ the componentwise floor operator. The set $\Delta\mathbb{Z}^p$ is the set of the p-tuples of integers multiple of Δ. The symbol $\langle\cdot,\cdot\rangle$ denotes the standard Euclidean inner product and $\times$ stands for the standard Cartesian product. For a set U, $\mathbf{int}(U)$ denotes the interior of U. The double arrows notation $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^m$ indicates that F is a set-valued mapping with $F(x) \subset \mathbb{R}^m$. Throughout the paper, a.a. stands for *almost all* in a Lebesgue-measure sense. For a function $f : A \to B$, $\mathrm{rge} f := \{y \in B : \exists x \in A \text{ such that } y = f(x)\}$.

Preliminary definitions: In this paper we deal with differential inclusions in the form

$$\dot{x} \in F(x) \tag{1}$$

where $F(x) : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$. Consider the following definitions given mainly in Goebel, Sanfelice, and Teel (2012) and Lin, Sontag, and Wang (1996).

Definition 1. Let $\mathbb{I} \subset \mathbb{R}_{\geq 0}$ be an interval. Given $x_0 \in \mathbb{R}^n$, an absolutely continuous function $\phi : \mathbb{I} \to \mathbb{R}^n$ is said to be a solution to (1) from x_0, if $\phi(0) = x_0$, and $\dot{\phi}(t) \in F(\phi(t))$ for a.a. $t \in \mathbb{I}$.

Definition 2. A solution $\phi : \mathbb{I} \to \mathbb{R}^n$ to (1) from x_0 is said to be maximal, if there does not exist any other solution $\overline{\phi} : \overline{\mathbb{I}} \to \mathbb{R}^n$, with $\mathbb{I} \subset \overline{\mathbb{I}}$ and such that $\phi(t) = \overline{\phi}(t)$ for every $t \in \mathbb{I}$. Moreover, ϕ is said to be complete, if $\mathbb{I} = \mathbb{R}_{\geq 0}$.

Definition 3. A set $\mathcal{A} \subset \mathbb{R}^n$ is strongly forward invariant for (1), if every maximal solution ϕ to (1) is complete, and $\phi(0) \in \mathcal{A}$ implies $\mathrm{rge}\,\phi \subset \mathcal{A}$.

2. Problem statement

Consider the following continuous-time linear system with quantized input:

$$\begin{cases} \dot{x} = Ax + B\,\mathrm{q}(u) \\ x(0) = x_0 \end{cases} \tag{2}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$, $x_0 \in \mathbb{R}^n$ are respectively the state, the input of the system and the initial state. A, B are real matrices of suitable dimensions, and $\mathrm{q}(\cdot)$ is the uniform quantizer, which is described by the static nonlinear functions defined as

$$\mathrm{q}(u) := \Delta\,\mathrm{sign}(u)\left\lfloor \frac{|u|}{\Delta} \right\rfloor \tag{3}$$

where Δ is a positive given real scalar characterizing the quantization error bound. Assuming that the state x is fully accessible, we want to stabilize system (2) via the following control law $u = Kx$. Therefore, by defining the function $\Psi(u) := \mathrm{q}(u) - u$, the closed-loop system reads as

$$\begin{cases} \dot{x} = (A + BK)x + B\Psi(Kx) \\ x(0) = x_0. \end{cases} \tag{4}$$

Notice that, due to the presence of the uniform quantizer, the right-hand side of (4) is a discontinuous function of the state, then there is no guarantee about the existence of solutions when intended in a classical sense; see Cortés (2008). To this end, as in Ceragioli and De Persis (2007), in this paper we focus on Krasovskii solutions to system (4), that is the solutions to the following differential inclusion:

$$\dot{x} \in \mathcal{K}\left((A + BK)x + B\Psi(Kx)\right) \tag{5}$$

where the Krasovskii operator $\mathcal{K}$ is defined by $\mathcal{K}(f(x)) := \bigcap_{\delta>0} f(\mathcal{B}(x,\delta))$. The existence of such solutions is guaranteed under the very mild requirement of local boundedness of the right-hand side of (4), obviously verified in our case. Moreover, as pointed out in Goebel et al. (2012) and Hájek (1979), Krasovskii solutions are arbitrarily close to the solutions to (4) obtained by perturbing the state x with arbitrarily small perturbations; see Hájek (1979). This fact provides a further reason to consider Krasovskii solutions, beyond the merely issue concerning the existence of solutions.

Remark 1. As pointed out in Liberzon (2003) and Tarbouriech and Gouaisbaut (2012), the presence of the uniform quantizer defined in (3), due to its deadzone effect, can represent a real obstacle to the asymptotic stabilization of the closed-loop system. Namely, one should be aware that if the matrix A is not Hurwitz, then the asymptotic stability of the origin for the closed-loop system (4) cannot be achieved via any choice of the gain K.

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