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State observers in the design of eigenstructures for enhanced sensitivity

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ABSTRACT

The problem of closed-loop enhanced sensitivity design is as follows: Given a linear time invariant system, find a (realizable) feedback gain such that: (1) the closed-loop is stable in the reference and the potentially damaged states, and (2) the eigenstructure includes a subset of poles, with desirable derivatives, that lie in a part of the plane where identification is feasible. This paper shows that pole derivatives with respect to system parameters for a controller/observer system, contrary to the assumption often made, depend on both the controller and the observer gains, i.e. the separation principle holds for placing the poles but does not extend to the pole derivatives. Closed-form expressions for the derivatives with due consideration to both gains are presented. Examination shows that the sum of these derivatives is independent of both gains, is constant along the nonlinear paths traced by the poles as damage increases and, provided the damage affects only the stiffness, is nearly zero.

1. Introduction

Eigenstructure assignment is a well-known scheme in which a static linear gain that satisfies design objectives is obtained by directly specifying closed loop poles and eigenvectors. Eigenstructures that, in addition to satisfying some performance objectives are least sensitive to uncertainties in some parameters are also of interest, and work related to this goal can be found in [1–5], among others. The flip side of sensitivity minimization, sensitivity maximization, has been proposed in Structural Health Monitoring to improve the resolution of damage characterization from identified eigenvalue shifts [6–10]. This paper examines how the presence of an observer in the loop, necessary when the operating mode is estimated state feedback, affects the sensitivities. Eigenvalue sensitivities depend on the right and left side eigenvectors of the controller/observer system and in this regard design for sensitivity enhancement is an eigenvector placement problem. Pole locations remaining relevant, however, since apart from the constraints imposed on them by identifiability and stability, their positions determine the subspaces wherein the closed-loop eigenvectors must lay [11].

The design of closed-loop eigenstructures for monitoring requires decisions on the cost function to be maximized (minimized), decisions on the extent and distribution of damage for which closed-loop stability must be satisfied, and specification of the limits the hardware imposes on the controller. While specific choices on these items are made in the numerical example, the objective of this paper is not to propose design criteria but to present consistent expressions for the evaluation of the derivatives of the poles of a controller/observer system. In as far as these derivatives go the common practice has been to assume that the separation principle, which holds true for pole positions, extends to the derivatives and, therefore, that

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results for the controller poles do not depend on the observer gain. It is shown here that this is not so, and while the reason is best appreciated in the context of the derivations, the essence can be stated from the outset, namely: pole derivatives depend on both gains because parameter shifts resulting from damage are not known to the observer and, as a consequence, pole positions in the perturbed state depend on both gains. It's opportune to note that the problem of robust pole placement in the presence of an observer [3,4] differs from the one considered here in that the issue in the former is how to select gains such that the deviations between the target and the realized controller/observer poles from inevitable model error are minimized; the focus here being, instead, on how small changes in parameters, taking place after the controller/observer is formulated, translate into movement of the poles.

In this paper we parameterize the controller/observer transition matrix to reflect the fact that the observer is unaware of changes due to damage and derive the consistent closed form expressions for the pole derivatives. Comparison between the expressions obtained and the ones that hold for full state measurements show that the effect of the observer is captured by a matrix that is the stabilizing solution of a Sylvester equation [12]. Numerical results show that the effect of the observer on the Jacobian of the controller poles, and on the extent of damage for which the closed-loop is stable, can be large. A number of lemmas extracted from the derivation are presented and proved; the most significant, given its implications in the design for stability, shows that the sum of the discrete time (DT) derivatives is independent of the controller and of the observer gains. A numerical section exemplifying the analytical examinations is included.

2. Sensitivity of controller/observer system

Let S be a non-defective but otherwise arbitrary square matrix that is a function of some parameter, θ. We shall refer to the derivatives of the eigenvalues of S with respect to θ as sensitivities; not to be confused with the matrix in the Laplace domain that carries the same name [13,14]. The sensitivity of the jth eigenvalue with respect to θ writes

$$\lambda_j' = \varphi_j^T S' \psi_j \tag{1}$$

where ψ_j and φ_j are the right and left side eigenvectors and the prime indicates differentiation with respect to θ. We now particularize Eq. (1) to the case where S is the transition matrix of a finite dimensional linear time invariant system under estimated state feedback. We begin with the expression that governs the evolution of the state in DT, namely

$$x_{k+1} = A_d x_k + B_d u_k + B_\omega \omega_k + B_f f_k \tag{2}$$

where $A_d \in R^{N\times N}$ is the transition matrix, $B_d, \in R^{N\times r}$ and $B_\omega \in R^{N\times z}$ are the control input and disturbance influence matrices, $u_k \in R^{r\times 1}$ are the control inputs and $\omega_k \in R^{z\times 1}$ the disturbances, which are typically assumed to be zero mean, Gaussian, and white, with covariance Q. Finally, if there are any deterministic exogenous excitations, $f_k \in R^{h\times 1}$, then $B_f \in R^{N\times h}$ is the associated influence matrix. Given the stochastic disturbances (plus the measurement noise) the poles identified from finite length signals are random variables and, consequently, so are the identified pole movements due to plant parameter changes (damage in this application). The expressions derived next are deterministic and correspond to the expectation level from an unbiased identification. With $K \in R^{N\times m}$ as the time invariant gain of the observer, the evolution of the estimated state is governed by

$$\hat{x}_{k+1} = A_d \hat{x}_k + B_d u_k + B_f f_k + K(y_k - \hat{y}_k) \tag{3}$$

where the true and the estimated outputs are denoted y_k and $\hat{y}_k \in R^{m\times 1}$. For full estimated state feedback we have

$$u_k = -G \cdot \hat{x}_k \tag{4}$$

where $G \in R^{r\times N}$ is the control gain, $\hat{x}$ is the estimated state and the minus sign is, of course, conventional. We restrict the output y_k to be a linear combination of the state plus some measurement noise, v_k, typically assumed zero mean, Gaussian, and white, with covariance R. Excluding cases with direct transmission, or assuming the direct transmission contribution is subtracted from the output, one has

$$y_k = C_d x_k + v_k \tag{5}$$

From previous results it follows that the state and the estimated state form a 2N linear system having the state space recurrence

$$\begin{Bmatrix} x_{k+1} \\ \hat{x}_{k+1} \end{Bmatrix} = \begin{bmatrix} A_d & -B_d G \\ K C_d & A_{do} - K C_d - B_d G \end{bmatrix} \begin{Bmatrix} x_k \\ \hat{x}_k \end{Bmatrix} + \begin{bmatrix} B_f \\ B_f \end{bmatrix} \{f_k\} + \begin{bmatrix} B_\omega & 0 \\ 0 & K \end{bmatrix} \begin{Bmatrix} \omega_k \\ v_k \end{Bmatrix} \tag{6}$$

where the reader will note that we've introduced notation to distinguish between the transition matrix that reflects changes due to damage, A_d, and the invariant transition matrix of the state estimator, A_{do}. Needless to say, in the reference state $A_{do} = A_d$. Observations on the poles and eigenvectors of the system in Eq. (6) are most easily made after introducing the basis transformation