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Short communication

Comparison of viscous and structural damping models for piezoelectric vibration energy harvesters

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This work compares viscous and structural damping in a lumped-parameter model of a piezoelectric vibration energy harvester. The dynamic response and power harvested are solved in closed-form for devices with combined viscous and structural damping. The results are then reduced to get expressions for isolated cases of viscous and structural damping. The conditions that maximize the power harvested are determined. These devices generally have two maxima. One maximum is not sensitive to the damping model. The other maximum, however, has meaningful differences between viscous and structural damping models. The differences between the two models increase with increasing electromechanical coupling.

1. Introduction

Analytical models for piezoelectric vibration energy harvesting devices commonly include viscous mechanical damping to account for the dissipative losses in the system. Lumped-parameter models in Refs. [1–13] include viscous damping. References [1,14–26] include viscous damping when analyzing piezoelectric beam distributed-parameter models. Viscous and strain rate damping are included in distributed-parameter models for piezoelectric beam devices in Refs. [27–30]. None of the references in our bibliography use a structural damping model, despite its general popularity for vibration analyses.

Many vibration energy harvester models use viscous damping for convenience and no obvious viscous elements are present in the physical device. The damping in these devices could come for the elastic structure itself, and elastic structures in sinusoidal vibration may not exhibit viscous damping behavior [31,32]. A structural damping model could be appropriate for some devices.

The choice of damping model is specific to a particular device and its operating environment. It is unlikely that the damping in all piezoelectric devices can be accurately modeled using a single, unique model. Careful experiments are necessary to determine whether a viscous, structural, combination of viscous and structural, or another damping model is appropriate for a particular device.

This technical note determines closed-form expressions for the dynamic response and average power harvested by piezoelectric vibration energy harvesters with viscous and structural damping models. Predictions of the power harvested by devices with viscous damping are compared to those with structural damping. Numerical results are presented for a wide range of damping and electromechanical coupling coefficients. Surprisingly-large differences are shown between the two damping models, even though these devices have purposely-low damping.

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2. Analytical model

A schematic of a lumped-parameter piezoelectric vibration energy harvester is shown in Fig. 1. The device consists of a proof mass m that is supported by a piezoelectric structure with stiffness k_p, electromechanical coupling e_p, capacitance C_p, and structural loss factor d_s. For most devices, these parameters depend on the properties of the piezoelectric material and the device's configuration and geometry. Deformations of the piezoelectric structure result in generated voltages $V(T)$ that power an electrical load with equivalent resistance R. The device is dynamically excited by vibration of its base with the form $Y(T) = Y_0 \cos \Omega T$, where Y_0 and Ω are its amplitude and frequency, respectively. The displacement $U(T)$ of the proof mass is relative to this vibration.

The nondimensional equations of motion for the device are

$$\ddot{u} + 2\zeta\dot{u} + (1 + j2\eta)u + \gamma v = \omega^2 \cos \omega t, \quad \alpha\dot{v} - \alpha\dot{u} + v = 0, \tag{1}$$

where the overdot is nondimensional time differentiation. Viscous mechanical damping, with viscous damping coefficient d_v, is added to Eq. (1) even though there are no viscous damping elements in the model. The nondimensional parameters in Eq. (1) are related to the dimensional ones by

$$u = U/Y_0, \quad v = C_p V/e_p Y_0, \quad t = \sqrt{k_p/m}T, \quad \omega = \Omega\sqrt{m/k_p}, \quad \eta = d_s/2k_p, \quad \zeta = d_v/2\sqrt{k_p m}, \quad \gamma = e_p^2/k_p C_p$$
$$\alpha = RC_p\sqrt{k_p/m}. \tag{2}$$

The model described by Eq. (1) applies to both piezoelectric stack devices and single-mode representations of piezoelectric beam devices. Lumped-parameter modeling of piezoelectric beam devices is discussed in Refs. [33,34]. Expressions for the stiffness, electromechanical coupling, and capacitance of bi-morph piezoelectric beams are given in Ref. [35]. The nondimensional damping coefficient ζ is the device's percent damping. The parameter η for the structural damping model has an analogous interpretation.

The closed-form, steady state solution to Eq. (1) is

$$u(t) = A\cos(\omega t - \phi), \quad v(t) = B\cos(\omega t - \psi), \tag{3a}$$

$$A = \frac{\omega^2\sqrt{1 + \alpha^2\omega^2}}{\sqrt{[1 - \omega^2 - \alpha\omega(2\eta + 2\zeta\omega)]^2 + [2\eta + 2\zeta\omega + \alpha\omega(1 + \gamma - \omega^2)]^2}}, \tag{3b}$$

$$\tan\phi = \frac{(2\eta + 2\zeta\omega)(1 + \alpha^2\omega^2) + \alpha\gamma\omega}{1 - \omega^2 + \alpha^2\omega^2(1 + \gamma - \omega^2)}, \tag{3c}$$

$$B = \frac{\alpha\omega^3}{\sqrt{[1 - \omega^2 - \alpha\omega(2\eta + 2\zeta\omega)]^2 + [2\eta + 2\zeta\omega + \alpha\omega(1 + \gamma - \omega^2)]^2}}, \tag{3d}$$

$$\tan\psi = \frac{\omega^2 - 1 + \alpha\omega(2\eta + 2\zeta\omega)}{2\eta + 2\zeta\omega + \alpha\omega(1 + \gamma - \omega^2)}. \tag{3e}$$

The power harvested by the device is $p = v^2/\alpha = B^2[1 + \cos(2\omega t - 2\psi)]/2\alpha$. Consistent with the literature, the performance of the device is quantified by the average power harvested over one oscillation cycle $\tau = 2\pi/\omega$. Averaging p in this way gives

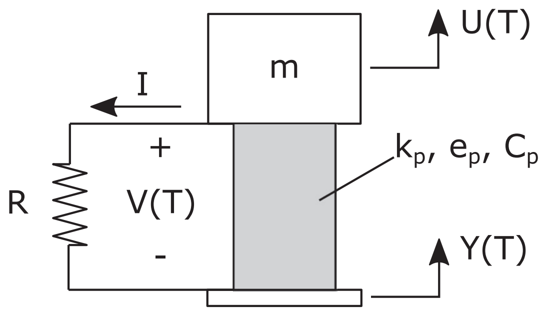

Fig. 1. Schematic of a piezoelectric vibration energy harvesting device.