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Non-linear vibrations of a beam with non-ideal boundary conditions and uncertainties – Modeling, numerical simulations and experiments

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A B S T R A C T

This paper presents experiments and numerical simulations of a nonlinear clamped-clamped beam subjected to Harmonic excitations and epistemic uncertainties. These uncertainties are propagated in order to calculate the dynamic response of the nonlinear structure via a coupling between the Harmonic Balance Method (HBM) and a non-intrusive Polynomial Chaos Expansion (PCE). The system studied is a clamped-clamped steel beam.

First of all, increasing and decreasing swept sine experiments are performed in order to show the hardening effect in the vicinity of the primary resonance, and to extract the experimental multi-Harmonic frequency response of the structure. Secondly, the Harmonic Balance Method (HBM) is used alongside a continuation process to simulate the deterministic response of the nonlinear clamped-clamped beam. Good correlations were observed with the experiments, on the condition of updating the model for each excitation level. Finally, the effects of the epistemic uncertainties on the variability of the nonlinear response are investigated using a non-intrusive Polynomial Chaos Expansion (PCE) alongside the Harmonic Balance Method (HBM). A new methodology based on a phase criterion was developed in order to allow the PCE analysis to be performed despite the presence of bifurcations in the nonlinear response. The efficiency and robustness of the proposed methodology is demonstrated by comparison with Monte Carlo simulations. Then, the stochastic numerical results are shown to envelope the experimental responses for each excitation level without the need for model updating, validating the nonlinear stochastic methodology as a whole.

1. Introduction

The vibration response of mechanical systems used to be mainly studied by means of a linear analysis. Indeed, numerical simulations for both the modal analysis and frequency response function of linear systems are implemented in every Finite Element software package and are widely used in industry. However, experimentally, the dynamical system can experience nonlinear behavior with a frequency response that strongly depends on the excitation level. The nonlinearities may be

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caused by large displacements, contact, friction in the joints, or non-elastic compounds. The consequences of such nonlinearities are a dependency of eigenfrequencies and dissipation with input amplitude, discontinuities in the frequency response and a multi-Harmonic response to a mono-Harmonic excitation.

Both the efficient modeling of the nonlinear behavior of mechanical systems and the development of nonlinear computational techniques are essential in order to proceed with an efficient and quick analysis of complex problems. Various computational methods have been developed to compute the nonlinear differential equations, and used over a wide range of mechanical engineering problems [1–3]. Among these, the Incremental Harmonic Balance Method [4] enables the multi-Harmonic frequency response of a nonlinear system to be computed. This method was implemented in a previous work [5] and compared with the method of multiple scales and the shooting method for the computation of the nonlinear response of the mechanical system used in this paper. It was illustrated that the Harmonic Balance Method gave excellent results in terms of both precision and computation time.

Moreover, it is nowadays recognized that the effects of uncertainties on the nonlinear dynamics response of mechanical systems remain a major issue in engineering applications. Several approaches have been developed to estimate the stochastic dynamic response of mechanical systems: classical perturbation methods [6], the Monte-Carlo simulations [7], or Polynomial Chaos Expansion [8] for instance.

In the present study, we propose to investigate the nonlinear behavior of a clamped-clamped beam with large displacements, that is modeled by a Duffing oscillator. This system has been studied in the previous work by Claeys et al. [5] without considering uncertainties. This previous work brought to light the dispersion of the results with respect to the excitation level, and thus the need to take into account the uncertainties. Results from the deterministic nonlinear numerical simulations are compared with those from various experiments. The mandatory model updating for each experiment enforces the need for an epistemic uncertainties propagation strategy. These uncertainties are due to a lack of knowledge or incomplete information. The primary objective of the present study is the validation of an extended polynomial chaos expansion, in order to be able to predict the stochastic response of the beam despite the presence of bifurcations in the vicinity of the resonance.

The paper is divided into five parts. Firstly, a brief description of the test structure and the analysis of various experimental data from vibrational tests on a bench in the CEA laboratory are presented. Secondly, the paper focuses on the modeling of the beam. The nonlinear simulation based on the well-known Harmonic Balance Method and the non-intrusive polynomial chaos expansion procedure are detailed in Section 4. Finally, results from the stochastic nonlinear numerical simulations are compared with those from a numerical reference (Monte-Carlo simulations) and the experiments. This last part illustrates the originality and peculiarities of the study, which proposes to build a non-intrusive polynomial chaos expansion for nonlinear vibrational systems with return points based on a phase criterion, leading to an increase in the confidence in the numerical models currently used with model uncertainties. The use of the phase criterion was proposed in previous works for linear [9] and nonlinear [10] systems subjected to uncertainties, using an intrusive approach and showing promising results.

2. Experiments

2.1. Experimental setup

The setup has been presented with minor changes relative to that presented in [5] for the same structure. The system studied is a steel beam of dimensions $470 \times 20 \times 5$ mm presented in Fig. 1 along with its instrumentation. The complete structure consists of the beam plus two heavy steel blocks of dimensions $100 \times 100 \times 85$ mm each. The entire piece is manufactured from a single bulk piece of steel with a progressive link between the beam and the blocks in order to limit the stress concentrations in the clamped area.

The blocks are bolted onto a large circular aluminum plate, itself bolted onto the vibrating pot. The system is instrumented with 4 three-axial accelerometers. Two accelerometers ($P1$ and $P2$) are positioned on the top of the blocks, one is placed at the center of the beam ($A1$), and one is placed on the plate ($A2$). Each signal is denoted by the sensor name followed by the direction of measurement.

The entrance signal $A2Z$ is the shaker's feedback control signal. The reference frame of the study is the circular plate, where $A2$ is fixed. The response signal is the acceleration at the middle of the beam, in the reference frame of the plate, that is $A1Z - A2Z$. The signals $P1Z$ and $P2Z$ help us to control the symmetry of the entrance signal.

2.2. Experimental results and discussion

The shaker is piloted with increasing and decreasing swept sines with a linear sweep rate. The excitation frequency evolves slowly over time (0.1 Hz/s in the vicinity of the resonance) to ensure that the response is stationary. Experimental results for three levels of excitation are plotted in Fig. 2. The shape is typical of a Duffing oscillator.

The shaker was set to limit the lowest experimental excitation level to $1\ \mathrm{m \cdot s^{-2}}$ because of the noise. The upper value was limited to $10\ \mathrm{m \cdot s^{-2}}$ in order to avoid the plasticizing of the beam. The experimental multi-Harmonic input/output is presented in Fig. 2 for a single level of excitation ($4\ \mathrm{m \cdot s^{-2}}$). The processing signal tool developed in [5] allows the identification