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Receptance-based stability criterion for second-order linear systems with time-varying delay

Tito L.M. Santos [a], José Mário Araújo [b,*], Taniel S. Franklin [a]

[a] Departamento de Engenharia Elétrica e de Computação, Universidade Federal da Bahia, Escola Politécnica, Rua Aristides Novis, 02, Federação, Brazil
[b] Grupo de Pesquisa em Sinais e Sistemas, Instituto Federal da Bahia, Rua Emídio dos Santos, S/N, Barbalho, 40301–015, Brazil

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A B S T R A C T

This paper presents a simple receptance-based criterion to analyze the robust stability of second-order systems with time-varying delay. The proposed approach is based on the closed-loop receptance which is directly related to the open-loop one by using the Sherman-Morrison equation. In this kind of problem, stability analysis cannot be performed from the closed-loop eigenvalues due to the time-variant nature of the system. In this context, a new robust stability condition is proposed by using the Small-Gain Theorem for second-order systems with either single or multiple inputs. The main contribution can be interpreted as a receptance-based generalization of the Single-Input Single-Output (SISO) first order Small-Gain Theorem condition. Moreover, the proposed stability criterion is combined with a detuning strategy to deal with the trade-off between performance and robustness with respect to delay variation. No limitation is imposed to the time-varying delay derivative which is a general result. Moreover, the proposed approach can also be used to analyze delay uncertainty due to the implementation simplicity since closed-loop poles are not computed in this criterion. Numerical examples are given to illustrate the effectiveness of the proposed approach.

1. Introduction

Closed-loop control of second-order systems has received increasing attention due to its relevance in practice [1–4]. Second-order models can be used to naturally represent real control problems such as active control of mechanical vibrations, electrical networks oscillations, vibroacoustic phenomena, among others [5–8]. In this kind of system, state feedback control is implemented by using state measurements and its derivative which provide interesting properties with respect to analysis and synthesis purposes [9–15]. The benefits of the second-order based approaches have been reported in remarkable related works [16–18]. In this context, receptance based design [17,18] is useful since the overall model identification does not require the knowledge of specific parameters [17,18,2]. Moreover, frequency response identification can be directly obtained from experimental data-set which simplifies identification procedure [17,18,2]. Despite the maturity with respect to the state feedback design for second-order linear systems, time-delay effect may cause significant performance degradation or even instability [19–21,4]. Stability analysis is more involved with time delay effect due to the transcendental description [22]. Anyway, closed-loop eigenvalues and stability analysis can be performed if the time delay is assumed to be constant [23–28]. However, robust stability analysis is an open problem with respect to second-order system with

* Corresponding author.
E-mail addresses: tlsantos@ufba.br (T.L.M. Santos), araujo@ieee.org (J.M. Araújo), tanielfranklin@gmail.com, taniel.franklin@fieb.org.br (T.S. Franklin).

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time-varying delay since the notion of poles and eigenvalues cannot be directly applied to time-variant systems. Note that the delay variation may introduce a neglected time-variant dynamic by itself which may be significantly different than the constant delay case.

Time delay effect naturally appears in real control problems due to the interval necessary to process and to transmit the information [1]. Constant time delay assumption is reasonable for most of the second-order control systems because delay variation is not significant in general. However, the delay variation may be an important issue in some applications due to the intrinsic behavior of the system [3] or as a consequence of the network induced delay [29] for instance. In general, time-varying delay effect is analyzed by means of first-order approaches even in the context of second-order systems [3,30] which is an undesired solution due to the well known benefits of the second order design and analysis concepts.

Several robust criteria have been proposed to analyze the stability of first-order system with time-varying delay [31–35]. These results are mainly based on Lyapunov-Razumikhin or Lyapunov-Krasovskii function (functionals) which require a first-order state-space description and the solutions are obtained from a Linear Matrix Inequalities (LMI) problem. As previously pointed, augmented description has several drawbacks such as enlarged dimension, numerical conditioning problems and some special properties cannot be easily explored such as no spillover projects. Moreover, receptance advantages such as identification and analysis simplicity cannot be applied.

The Small-Gain Theorem provides an interesting robust stability criterion due to its implementation and conceptual simplicity [31]. This idea was explored in [31] to deal with Single-Input Single-Output (SISO) systems with time-varying delay. This result is a remarkable exception since continuous-time and discrete-time criteria are based on SISO transfer functions. The extension for multivariable cases was presented in [32], but the analysis is formulated as an Integral Quadratic Constraint problem which depends on a first-order description with LMI solution. Thus, the ideas of [31] cannot be directly applied to second-order systems and [32] has the same drawbacks of the Lyapunov based solutions.

This paper proposes a simple stability criterion for second-order systems with time-varying delay based on the receptance approach. Inspired by the ideas of [31], the Small-Gain Theorem for systems with time-varying delay is extended for second-order systems with multiple inputs. The maximal allowable time-varying delay is obtained from the closed-loop receptance. Sherman-Morrison formula is used to relate stability criterion with open-loop receptance in order to obtain a simplified analysis tool. Moreover, a detuning approach is proposed to potentially increase the allowable delay if necessary. The receptance-based stability criterion can also be used to deal with unknown constant delay due to its simplicity since the analysis is based on frequency information instead of closed-loop eigenvalues. Three case studies are presented to illustrate the usefulness of the proposed approach.

The paper is organized as follows: Section 2 introduces the problem statement, Section 3 presents the main results, a detuning strategy is proposed in Section 4, the case studies are analyzed in Section 5, and the concluding remarks are drawn in Section 6.

2. Preliminaries

The second-order linear system without delay is initially described by:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{B}\mathbf{u}(t). \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are, respectively, the mass, damping and stiffness matrices, each of order $n, \mathbf{x}(t) \in \mathbb{R}^n$ is the displacement vector, $\mathbf{B}\mathbf{u}(t)$ represents the external force and $\mathbf{u}(t) \in \mathbb{R}^p$. Moreover, it is assumed that $\mathbf{M} \in \mathbb{R}^{n\times n}$ is a non-singular matrix ($\det(\mathbf{M}) \neq 0$). The open-loop schematic representation of a second-order systems without delay is shown in Fig. 1 where $\mathbf{x}(t)$ and $\dot{\mathbf{x}}(t)$ are measured variables available for control purposes.

The open-loop receptance (or simply receptance) is represented by:

$$\mathbf{R}(s) = (\mathbf{M}s^2 + \mathbf{C}s + \mathbf{K})^{-1} \tag{2}$$

where s is the Laplace complex variable. This transfer function matrix is related to an important design approach since the second-order linear model can be directly obtained from an experimental data-set. Moreover, the control law can be defined from the receptance information which is useful for high-dimension systems since the augmented state-space description is not necessary. For the sake of presentation purposes, note that the second order model without delay can be alternatively described by:

$$\mathbf{P}(s) = \begin{bmatrix} \mathbf{R}(s)\mathbf{B} \\ s\mathbf{R}(s)\mathbf{B} \end{bmatrix} \tag{3}$$

where $\mathbf{z}(t) = [\mathbf{x}(t)^T \ \dot{\mathbf{x}}(t)^T]^T$ by definition, $\mathbf{Z}(s) = \mathcal{L}\{\mathbf{z}(t)\}$, $\mathbf{U}(s) = \mathcal{L}\{\mathbf{u}(t)\}$, $\mathbf{Z}(s) = \mathbf{P}(s)\mathbf{U}(s)$ and $\mathcal{L}\{\cdot\}$ denotes the Laplace transform of a given signal. In general, the control law is defined by:

$$\mathbf{u}(t) = \mathbf{F}\dot{\mathbf{x}}(t) + \mathbf{G}\mathbf{x}(t) \tag{4}$$

where $\mathbf{F} \in \mathbb{R}^{p\times n}$ and $\mathbf{G} \in \mathbb{R}^{p\times n}$ are feedback gain matrices.