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Design of continuous–discrete observers for time-varying nonlinear systems☆

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ABSTRACT

We present a new design for continuous–discrete observers for a large class of continuous time nonlinear time-varying systems with discrete time measurements. Using the notion of cooperative systems, we show that the solutions of the observers converge to the solutions of the original system, under conditions on the nonlinear terms and on the largest sampling interval. Our conditions are given by explicit expressions.

1. Introduction

In real world applications, the state variables may be difficult to measure. Such applications can often be modeled using systems with outputs. Then one builds an observer for the state such that the observation error between the observer value and the state value converges to 0 as time goes to ∞. Much of the observers literature is under continuous measurements. See, e.g., Zemouche, Boutayeb, and Bara (2008), which gives observers under continuous state measurements, by expressing the differential equation satisfied by the estimation error in terms of a linear parameter varying system.

However, in many engineering applications, measurements are collected at discrete times. This produces continuous–discrete systems, where the dynamics are continuous time but the output measurements are only available at discrete instants. There is a large literature, spanning over 40 years, on ways to build observers for continuous–discrete systems. See, e.g., Jazwinski (2007), which used a continuous–discrete Kalman filter to solve a filtering problem for stochastic continuous–discrete time systems.

The high gain observer in Gauthier, Hammouri, and Othman (1992) was adapted to continuous–discrete systems in Deza, Busvelle, Gauthier, and Rakotopara (1992), where the correction gain of the impulsive correction is obtained by integrating a continuous–discrete time Riccati equation. The robustness of observers with respect to discretization was studied in Arcak and Nesic (2004). See also Ahmed-Ali, Van Assche, Massieu, and Dorleans (2013), Farza et al. (2013) and Karafyllis and Kravaris (2009) for observers based on output predictors and Andrieu and Nadri (2010), Deza et al. (1992), Hammouri, Nadri, and Mota (2006), Karafyllis and Kravaris (2012), Mazenc and Dinh (2013, 2014), Tellez-Anguiano et al. (2012); and see Ahmed-Ali, Karafyllis, and Lamnabhi-Lagarrigue (2013), which presents results that allow delayed and sampled measurements. The work Ahmed-Ali, Postoyan, and Lamnabhi-Lagarrigue (2009) builds continuous–discrete observers for nonlinear systems, where the input acts on the system to satisfy a persistent excitation condition, while Nadri and Hammouri (2003) covers systems with known inputs and which are

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linear in the state. The work Karafyllis and Kravaris (2009) shows that if a system admits a suitable continuous time observer and the observer satisfies certain robustness properties, then one can augment the observer by a new output predictor system to produce a continuous–discrete observer. Also, Karafyllis and Kravaris (2009) shows how this observer augmentation process applies to key classes of linear and triangular globally Lipschitz systems. See Remark 3 for more discussions on Astorga, Othman, Othman, Hammouri, and McKenna (2002); and Karafyllis and Kravaris (2009) for continuous–discrete observers for an important model of emulsion polymerization reactors.

Here, we revisit Andrieu and Nadri (2010). We present a new construction of continuous–discrete observers (which are also called hybrid observers in the literature) for continuous time Lipschitz systems with discrete time measurements. Following the approach in Andrieu and Nadri (2010) and Deza et al. (1992), our continuous–discrete observer is obtained in two steps. First, when no measurement is available, the state estimate is computed by integrating the model. Then, when a measurement occurs, the observer makes an impulsive correction to the estimate. The work Andrieu and Nadri (2010) and Dinh, Andrieu, Nadri, and Serres (2015) used this type of algorithm to show that when no measurement occurs, the estimation error is a solution to an unknown linear parameter varying system. This gave a continuous–discrete analog of the continuous time measurement approach from Zemouche et al. (2008), and makes it possible to build a set that is guaranteed to contain all relevant solutions for all nonnegative times. Using this set, certain correction terms are designed to ensure that the estimation error asymptotically converges to zero. However, Andrieu and Nadri (2010) and Dinh et al. (2015) find the set by integrating a system with commutation, which does not lead to an explicit analytic expression. This may be an obstruction to using this type of approach in applications. Here, we use tools that are inspired by Cacace, Germani, Manes, and Setola (2012), Haddad, Chellaboina, and Hui (2010), Mazenc and Dinh (2013) and Raissi, Efimov, and Zolghadri (2012). We obtain analytical methods for constructing sets that are guaranteed to contain the relevant trajectories. Our results are strong and may be better suited to applications, since we allow nonlinearities in the systems and because we prove robustness to perturbations in the sampling schedule.

In the next section, we provide definitions. In Section 3, we present our new results on framers, which are of independent interest. In Section 4, we use our new results on framers to prove our theorem on continuous–discrete observers. Our closed form formulas for the framers make it possible to check the assumptions using linear matrix inequalities. We illustrate our main result in Section 5, using a motor dynamics and a pendulum system, which show how our approach can lead to a much larger maximal allowable measurement stepsize than was reported in Dinh et al. (2015). In Section 6, we summarize the value added by our work and suggest possible topics for follow-up research.

For a tutorial paper on the theory of continuous–discrete observers that states results from this and other recent papers (without giving their proofs and also without the examples we provide below) and also contains a generalization of Lemma 3, see Mazenc, Andrieu, and Malisoff (2015).

2. Notation, definitions, and basic result

Throughout the sequel, we omit arguments of functions when the arguments are clear from the context. We set $\mathbb{N} = \{1, 2, \ldots\}$. For any k and n in $\mathbb{N}$, the $k \times n$ matrix all of whose entries are 0 will also be denoted by 0, and we use $A = [a_{i,j}]$ to indicate that an arbitrary matrix $A \in \mathbb{R}^{k\times n}$ has $a_{i,j}$ in its ith row and jth column for each $i \in \{1, 2, \ldots, k\}$ and $j \in \{1, 2, \ldots, n\}$. The usual Euclidean norm of vectors, and the induced norm of matrices, of any dimensions are denoted by $|\cdot|$, and I is the identity matrix in the dimension under consideration. All inequalities and maxima must be understood to hold *componentwise*, i.e., if $A = [a_{i,j}]$ and $B = [b_{i,j}]$ are matrices of the same dimensions, then we use $A \le B$ to mean that $a_{i,j} \le b_{i,j}$ for all i and j, and $\max\{A, B\}$ is the matrix $C = [c_{i,j}]$ where $c_{i,j} = \max\{a_{i,j}, b_{i,j}\}$ for all i and j. A square matrix is called *cooperative* or *Metzler* provided all of its off-diagonal entries are nonnegative. Recall that the *Schur complement* of a symmetric matrix of the form

$$X = \begin{bmatrix} A & B \\ B^\top & C \end{bmatrix}$$

with an invertible matrix A is $S = C - B^\top A^{-1} B$, where $\top$ means the transpose. The following is well known: X is positive definite if and only if A and S are both positive definite. For each $r \in \mathbb{N}$ and each function $\mathcal{F} : [0, \infty) \to \mathbb{R}^r$, we set $\mathcal{F}(t_-) = \lim_{s\to t, s<t} \mathcal{F}(s)$ for all $t > 0$. For any matrices A and B in $\mathbb{R}^{n\times n}$, we use $A \preccurlyeq B$ (resp., $A \succ B$) to mean that $X^\top(A-B)X \le 0$ for all $X \in \mathbb{R}^n$ (resp., $X^\top(A-B)X > 0$ for all $X \in \mathbb{R}^n \setminus \{0\}$). Therefore, $\succeq$ has a different meaning from the partial order $\ge$ on matrices.

A system $\dot{x}(t) = f(t, x(t))$ whose solution is uniquely defined on $[t_0, \infty)$ for each initial condition $x(t_0)$ and each $t_0 \ge 0$ is called *nonnegative* provided that for each initial condition satisfying $x(t_0) \ge 0$, the solution $x(t)$ is nonnegative for all $t \ge t_0$. The following lemma is a direct consequence of Haddad et al. (2010, Proposition 2.2):

Lemma 1. *Consider any system of the form*

$$\dot{z}(t) = \mathfrak{A}(t)z(t) + \mathfrak{B}(t) \tag{1}$$

with state space $\mathbb{R}^n$, where $\mathfrak{A} : \mathbb{R} \to \mathbb{R}^{n\times n}$ and $\mathfrak{B} : \mathbb{R} \to \mathbb{R}^n$ are continuous. Assume that for all $t \ge 0$, the matrix $\mathfrak{A}(t)$ is Metzler and $\mathfrak{B}(t) \ge 0$. Then (1) is nonnegative.

3. Preliminary result on framers

In this section, we present preliminary results on framers for linear systems that we use in the next section to design our observers for nonlinear systems. We consider any linear time-varying system of the form

$$\dot{x}(t) = \mathfrak{M}(t)x(t) \tag{2}$$

with state space $\mathbb{R}^n$, where all entries of $\mathfrak{M} : [0, \infty) \to \mathbb{R}^{n\times n}$ are continuous. Let $\varrho : \mathbb{R}^2 \to \mathbb{R}^{n\times n}$ denote the fundamental solution of (2). Then, $\frac{\partial}{\partial t}\varrho(t, t_0) = \mathfrak{M}(t)\varrho(t, t_0)$ and $\varrho(t_0, t_0) = I$ hold for all $t_0 \ge 0$ and $t \ge t_0$. In this section, we provide componentwise lower and upper bounds for $\Gamma(t) = \varrho(t, 0)$. Notice for later use that the unique solution $\phi(\cdot, x_0)$ of the initial value problem

$$(\partial\phi/\partial t)(t, x_0) = \mathfrak{M}(t)\phi(t, x_0), \quad \phi(0, x_0) = x_0 \tag{3}$$

satisfies $\phi(t, x_0) = \Gamma(t)x_0$ for all $t \ge 0$ and $x_0 \in \mathbb{R}^n$.

3.1. Bounds for cooperative linear systems

We first present a preliminary result on framers, which we use in the next subsection to prove our main result on framers. Throughout this subsection, we assume:

Assumption 1. There are two constant Metzler matrices $\overline{\mathfrak{M}} \in \mathbb{R}^{n\times n}$ and $\underline{\mathfrak{M}} \in \mathbb{R}^{n\times n}$ such that

$$\underline{\mathfrak{M}} \le \mathfrak{M}(t) \le \overline{\mathfrak{M}} \tag{4}$$

hold for all $t \ge 0$. Also, $\mathfrak{M}$ is continuous.