Automatica 57 (2015) 21–28

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Output feedback stabilization of Boolean control networks[☆]

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ARTICLE INFO

Article history:
Received 1 April 2014
Received in revised form
29 January 2015
Accepted 16 March 2015
Available online 22 April 2015

Keywords:
Boolean control networks
Output feedback
System biology

ABSTRACT

In the paper output feedback control of Boolean control networks (BCNs) is investigated. First, necessary and sufficient conditions for the existence of a time-invariant output feedback (TIOF) law, stabilizing the BCN to some equilibrium point, are given, and constructive algorithms to test the existence of such a feedback law are proposed. Two sufficient conditions for the existence of a stabilizing time-varying output feedback (TVOF) are then given. Finally, an example concerning the *lac* Operon in the bacterium *Escherichia Coli* is presented, to illustrate the effectiveness of the proposed techniques.
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1. Introduction

Research in Boolean networks (BNs) and Boolean control networks (BCNs) has a long tradition. The renewed interest witnessed in recent times, however, must be mainly credited to two reasons: on the one hand, BNs and BCNs have proved to be effective modelling tools for a number of rapidly evolving research topics, like genetic regulation networks and consensus problems (Lou & Hong, 2010; Shmulevich, Dougherty, Kim, & Zhang, 2002). On the other hand, the algebraic framework developed by D. Cheng and co-authors (Cheng, 2009; Cheng & Qi, 2010; Cheng, Qi, & Li, 2011) has allowed to cast both BNs and BCNs into the framework of linear state-space models (operating on canonical vectors), thus benefitting of a number of powerful algebraic tools, in addition to traditional graph-based techniques. By resorting to this approach, many properties of BCNs such as the stability of an equilibrium point or a limit cycle, controllability, observability, and reconstructibility have been thoroughly investigated, and important control problems, such as state feedback stabilization, state-observer design, finite and infinite horizon optimal control have been solved (see, e.g. Cheng and Liu (2009), Cheng, Qi, Li, and Liu (2011), Fornasini and Valcher (2013a,b), Li, Yang, and Chu (2013)). The purpose of this contribution is to investigate output feedback control and to single out structural conditions that guarantee the existence of a stabilizing output feedback control law for a BCN. Static output feedback involves only memoryless operations on measurable quantities, and hence, when applicable, it provides an extremely simple and reliable tool. In particular, when modelling a biological regulation network, introducing a feedback loop from some specific (output) variables can be viewed either as an attempt to achieve a more complete representation of the network dynamics, since feedback loops are intrinsic of the network functioning, or as an external action aimed at modifying the network behaviour. The stabilizing output feedback problem has been addressed in Li and Wang (2013), where an algebraic characterization of a logical matrix that describes a stabilizing time invariant output feedback (TIOF) control law has been provided (Theorem 1). This condition is not computationally meaningful, as it does not provide a practical way to test whether a stabilizing output feedback law exists or not. On the other hand, the necessary and sufficient condition for output feedback stabilization provided in Theorem 2 of Li and Wang (2013) is actually only sufficient. The reason for this is that the state feedback matrices K the authors consider in order to obtain output feedback matrices K_y, by making use of the equation $K = K_y H$, are those obtained through the algorithm given in Li et al. (2013), an algorithm that provides only those state feedback matrices K that guarantee that each state reaches the equilibrium state, $\mathbf{x}_e$, along the shortest possible path. As we shall see in Example 4, in some cases stabilization to a given state $\mathbf{x}_e$ is achievable through state feedback, but not through output feedback. In other cases, a stabilizing output feedback does exist, but the associated state feedback law does not implement a shortest path strategy (see Example 1), that is always achievable when a state feedback is carefully designed. So stabilizing TIOF, when available, often achieves the stabilization to the equilibrium point at the price of a slower

[☆] The material in this paper was partially presented at the 2014 American Control Conference, June 4–6, 2014, Portland, OR, USA. This paper was recommended for publication in revised form by Associate Editor Yoshito Ohta under the direction of Editor Richard Middleton.

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http://dx.doi.org/10.1016/j.automatica.2015.03.032
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dynamics. In other words, there is no way of interpreting the associated control law as a shortest path state feedback law (Fornasini & Valcher, 2013b; Li et al., 2013).

The paper is organized as follows. In Section 2, we investigate under what conditions there exists a TIOF law, stabilizing a BCN to some equilibrium state $\mathbf{x}_e$. In Section 3, we provide sufficient conditions for the existence of stabilizing time-varying output feedback (TVOF) laws. Finally, Section 4 outlines the main features of the *lac* Operon model, and discusses the stabilization to a fixed point when TIOF or TVOF laws are implemented. A preliminary version of part of the results of Sections 2 and 3 appeared in Fornasini and Valcher (2014).

Notation. $\mathbb{Z}_+$ denotes the set of nonnegative integers. Given $k, n \in \mathbb{Z}_+$, with $k \leq n$, the symbol $[k, n]$ denotes the integer set $\{k, k+1, \ldots, n\}$. We consider Boolean vectors and matrices, taking values in $\mathcal{B} := \{0, 1\}$, with the usual operations (sum $\vee$, product $\wedge$ and negation $\neg$). δ_k^i denotes the ith canonical vector of size k, $\mathcal{L}_k$ the set of all k-dimensional canonical vectors, and $\mathcal{L}_{k\times n} \subset \mathcal{B}^{k\times n}$ the set of all $k \times n$ matrices whose columns are canonical vectors. Any matrix $L \in \mathcal{L}_{k\times n}$ can be represented as a row vector whose entries are canonical vectors, i.e. $L = [\delta_k^{i_1} \quad \delta_k^{i_2} \quad \ldots \quad \delta_k^{i_n}]$, for some indices $i_1, i_2, \ldots, i_n \in [1, k]$. The k-dimensional vector with all entries equal to 1 is denoted by $\mathbf{1}_k$. The (ℓ, j)th entry of a matrix M is denoted by $[M]_{\ell j}$, while the ℓth entry of a vector $\mathbf{v}$ is $[\mathbf{v}]_\ell$.

Given a matrix $L \in \mathcal{B}^{k\times k}$ (in particular, $L \in \mathcal{L}_{k\times k}$), we associate with it a *digraph* $\mathcal{D}(L)$, with vertices $1, \ldots, k$. There is an arc (j, ℓ) from j to ℓ if and only if $[L]_{\ell j} = 1$. A sequence $j_1 \to j_2 \to \cdots \to j_r \to j_{r+1}$ in $\mathcal{D}(L)$ is a *path* of length r from j_1 to j_{r+1} provided that $(j_1, j_2), \ldots, (j_r, j_{r+1})$ are arcs of $\mathcal{D}(L)$. A closed path is called a *cycle*. In particular, a cycle γ with no repeated vertices is called *elementary*, and its length $|\gamma|$ coincides with the number of (distinct) vertices appearing in it. There is a bijective correspondence between Boolean variables $X \in \mathcal{B}$ and vectors $\mathbf{x} \in \mathcal{L}_2$, defined by

$$\mathbf{x} = \begin{bmatrix} X \\ \neg X \end{bmatrix}.$$

$\mathbf{x}$ is called *the vector form* of the Boolean variable X. The *(left) semi-tensor product* $\ltimes$ between matrices $L_1 \in \mathbb{R}^{r_1\times c_1}$ and $L_2 \in \mathbb{R}^{r_2\times c_2}$ (in particular, $L_1 \in \mathcal{L}_{r_1\times c_1}$ and $L_2 \in \mathcal{L}_{r_2\times c_2}$) is defined as

$$L_1 \ltimes L_2 := (L_1 \otimes I_{T/c_1})(L_2 \otimes I_{T/r_2}), \quad T := \text{l.c.m.}\{c_1, r_2\},$$

where l.c.m. denotes the least common multiple. The semi-tensor product represents an extension of the standard matrix product, by this meaning that if $c_1 = r_2$, then $L_1 \ltimes L_2 = L_1 L_2$. If $\mathbf{x}_1 \in \mathcal{L}_{r_1}$ and $\mathbf{x}_2 \in \mathcal{L}_{r_2}$, then $\mathbf{x}_1 \ltimes \mathbf{x}_2 \in \mathcal{L}_{r_1 r_2}$. For the properties of the semi-tensor product we refer to Cheng, Qi, and Li (2011). By resorting to the semi-tensor product, we can extend the previous correspondence to a bijective correspondence between $\mathcal{B}^n$ and $\mathcal{L}_{2^n}$. Given $X = [X_1 \quad X_2 \quad \ldots \quad X_n]^\top \in \mathcal{B}^n$, just set

$$\mathbf{x} := \begin{bmatrix} X_1 \\ \neg X_1 \end{bmatrix} \ltimes \begin{bmatrix} X_2 \\ \neg X_2 \end{bmatrix} \ltimes \cdots \ltimes \begin{bmatrix} X_n \\ \neg X_n \end{bmatrix}.$$

2. Static output feedback control of BCNs

A *Boolean control network* (BCN) is described by the following equations

$$\begin{aligned} X(t+1) &= f(X(t), U(t)), \\ Y(t) &= h(X(t)), \quad t \in \mathbb{Z}_+, \end{aligned} \tag{1}$$

where $X(t)$, $U(t)$ and $Y(t)$ denote the n-dimensional state variable, the m-dimensional input and the p-dimensional output at time t, taking values in $\mathcal{B}^n$, $\mathcal{B}^m$ and $\mathcal{B}^p$, respectively. f, h are (logic) functions, i.e. $f : \mathcal{B}^n \times \mathcal{B}^m \to \mathcal{B}^n$ and $h : \mathcal{B}^n \to \mathcal{B}^p$. If we replace the state, input and output Boolean variables with their vector forms, namely with canonical vectors in $\mathcal{L}_N$, $N := 2^n$, $\mathcal{L}_M$, $M := 2^m$, and $\mathcal{L}_P$, $P := 2^p$, respectively, we can describe the BCN (1) by means of the following *algebraic representation* Cheng, Qi, and Li (2011):

$$\begin{aligned} \mathbf{x}(t+1) &= L \ltimes \mathbf{u}(t) \ltimes \mathbf{x}(t), \quad t \in \mathbb{Z}_+, \\ \mathbf{y}(t) &= H\mathbf{x}(t), \end{aligned} \tag{2}$$

where $\mathbf{x}(t) \in \mathcal{L}_N$, $\mathbf{u}(t) \in \mathcal{L}_M$ and $\mathbf{y}(t) \in \mathcal{L}_P$. $L \in \mathcal{L}_{N\times NM}$ and $H \in \mathcal{L}_{P\times N}$ are matrices whose columns are canonical vectors of size N and P, respectively. For every value δ_M^j of $\mathbf{u}(t)$, $L \ltimes \mathbf{u}(t) =: L_j$ is a matrix in $\mathcal{L}_{N\times N}$, and hence $L = [L_1 \quad L_2 \quad \ldots \quad L_M]$.

Before proceeding, we want to recall the state feedback stabilization problem. To this end we introduce the concepts of reachability and stabilizability.

Definition 1 (*Cheng, Qi, and Li, 2011*)**.** Given a BCN (2), $\mathbf{x}_f = \delta_N^j$ is *reachable* from $\mathbf{x}_0 = \delta_N^h$ if there exists $\tau \in \mathbb{Z}_+$ and an input $\mathbf{u}(t) \in \mathcal{L}_M$, $t \in [0, \tau - 1]$, that leads the state trajectory from $\mathbf{x}(0) = \mathbf{x}_0$ to $\mathbf{x}(\tau) = \mathbf{x}_f$.

$\mathbf{x}_f = \delta_N^j$ is reachable from $\mathbf{x}_0 = \delta_N^h$ if and only if Cheng, Qi, and Li (2011) there exists $\tau \in \mathbb{Z}_+$ such that the Boolean sum of the matrices L_i, $i \in [1, M]$, i.e. $L_{tot} := \bigvee_{i=1}^M L_i$, satisfies $[L_{tot}^\tau]_{jh} > 0$.

Definition 2 (*Cheng & Liu, 2009; Cheng, Qi, and Li, 2011; Fornasini & Valcher, 2013b*)**.** A BCN (2) is *stabilizable* to the state $\mathbf{x}_e \in \mathcal{L}_N$ if for every $\mathbf{x}(0) \in \mathcal{L}_N$ there exist $\mathbf{u}(t)$, $t \in \mathbb{Z}_+$, and $\tau \in \mathbb{Z}_+$ such that $\mathbf{x}(t) = \mathbf{x}_e$ for every $t \geq \tau$.

The characterization of stabilizability is immediate.

Proposition 1 (*Cheng, Qi, and Li, 2011; Fornasini & Valcher, 2013b; Li et al., 2013*)**.** *A BCN (2) is stabilizable to $\mathbf{x}_e \in \mathcal{L}_N$ if and only if the following conditions hold:*

(1) *$\mathbf{x}_e$ is an equilibrium point of the ith subsystem $\mathbf{x}(t+1) = L_i\mathbf{x}(t)$, for some $i \in [1, M]$, i.e. $\mathbf{x}_e = L \ltimes \delta_M^i \ltimes \mathbf{x}_e$;*
(2) *$\mathbf{x}_e$ is reachable from every initial state $\mathbf{x}(0)$.*

What is more interesting is the fact that if a BCN (2) is stabilizable to the state $\mathbf{x}_e$, then stabilization is achievable by means of a time-invariant state feedback $\mathbf{u}(t) = K\mathbf{x}(t)$ (Fornasini & Valcher, 2013b; Li et al., 2013). Furthermore, such a state feedback law can always be chosen in such a way that $\mathbf{x}_e$ is reached from every other state in a minimal number of steps. The question arises: under what conditions can we stabilize the BCN to $\mathbf{x}_e$ by resorting to an *output* feedback?

Definition 3. A BCN (2) is *TIOF stabilizable* to the state $\mathbf{x}_e \in \mathcal{L}_N$ if there exists $K_y \in \mathcal{L}_{M\times P}$ such that the output feedback law $\mathbf{u}(t) = K_y\mathbf{y}(t)$, $t \in \mathbb{Z}_+$, drives every $\mathbf{x}(0) \in \mathcal{L}_N$ to the state $\mathbf{x}_e$ in a finite number of steps, namely $\exists \tau \in \mathbb{Z}_+$ such that $\mathbf{x}(t) = \mathbf{x}_e$ for every $t \geq \tau$.

As for linear state-space models, the TIOF stabilization problem is easy to state, but quite challenging to be solved in a computationally tractable way. Clearly, if K_y defines a TIOF law, then $K = K_yH$ defines a state feedback law. So, a possible way could be that of determining whether the set of all stabilizing state feedback matrices includes at least one matrix expressed as $K = K_yH$ for some $K_y \in \mathcal{L}_{M\times P}$ (see Li and Wang (2013)). In general, the search cannot be restricted to the class of state feedback matrices K that implement paths of minimum length from each state to the equilibrium state $\mathbf{x}_e$ (Fornasini & Valcher, 2013b) and, consequently, the test has to be performed on the whole set of state feedback matrices (see Example 1).

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