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Direction cosine matrix estimation with an inertial measurement unit

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ABSTRACT

Estimating attitude using an inexpensive MEMS inertial measurement unit has many applications in smart phones, wearable sensors, rehabilitation medicine and robots. Traditional approaches to attitude estimation from the aerospace world focus on the use of either Euler angles or quaternions. These approaches suffer from disadvantages including singularities and nonlinear models. This paper proposes a method to estimate the direction cosine matrix (DCM) which encapsulates attitude information, instead of Euler angles or quaternions. The DCM does not suffer from singularities and also has linear dynamics. A rigorous DCM estimation algorithm, that incorporates automatic magnetometer bias calibration and satisfaction of an inherent orthonormal property of the DCM, is developed. The validity of the developed algorithm is demonstrated through experimental results with estimation of attitude on a 5-DOF robot. The estimation results are compared with values computed from encoders on the robot as well as with results from previously published algorithms.

1. Introduction

The attitude of an object is defined as the imaginary 3-D rotation that is needed to move the object from an initial reference orientation to its current orientation. Typically, the attitude is given relative to a frame of reference, usually specified by a Cartesian coordinate system. Accurate attitude information is essential in the controllers for many aerospace systems, such as satellites and unmanned aerial vehicles (UAVs). The rise of the smart phone and other wearable devices in recent years also requires attitude information for various applications, such as gesture recognition [1–3], video game controllers, rehabilitation [4], underwater robotic navigation [5] and assembly of parts [6]. Unfortunately, there exists no sensor that can directly measure attitude. Although the development of algorithms for attitude estimation is widely discussed in aerospace control literature, the sensors used for that purpose usually include multiple GPS receivers and other geological sensors, not just inertial sensors [7]. For robotic and daily life applications, GPS is often not usable. There exists a strong need to develop a systematic way to estimate the attitude of a rigid body using just an inexpensive inertial measurement unit (IMU), which includes accelerometers, gyroscopes and magnetometers.

A popular approach to attitude estimation is through integration of the angular rate measurements from a tri-axis gyroscope. The drifting effects due to gyroscope bias then need to be compensated by using additional measurement data from accelerometers and magnetometers. However, the formulation of the kinematic model with this approach and the

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corresponding estimator structure significantly vary in different papers. Since Euler angles are the most intuitive way to represent attitude information, several papers use them for some simple applications [2]. However, it is well known that Euler angles based estimation methods suffer from a singularity problem as the pitch angle approaches $\pm 90^\circ$ (gimble lock phenomenon). Therefore, the applications of this method are limited to the cases where the pitch motion is small. Another way to parametrize the attitude is the unit quaternion. In spite of complex quaternion algebra, it is the most popular way for attitude estimation in the navigation and aerospace societies [8]. Although the quaternion based approach solves the singularity problem, the inherent nonlinearity in the kinematic model for quaternions makes the estimator design quite challenging. The extended Kalman filter (EKF) or unscented Kalman filter (UKF) is a common tool used to perform nonlinear estimation [9,10,3,11] with quaternions. One prominent drawback of the EKF and UKF is that global convergence cannot be guaranteed, which may lead to a divergence in case of large deviations between the initial condition of the estimator and the true system.

This paper proposes the estimation of the direction cosine matrix (DCM) instead of the Euler angles or quaternions. As will be shown in later sections, the DCM provides a more intuitive way to represent the attitude than quaternions and the singularity problem that paralyzes the Euler angles based approach also disappears. Although the DCM is a matrix with nine elements, only six of them are independent. The existing DCM estimation methods in the literature either require extra information from expensive sensors [12] or incorporate a nonlinear kinematic model and a nonlinear Kalman filter, such as EKF, UKF [13,8,14,15] or nonlinear complementary filters [16]. This paper formulates the kinematics of the DCM as a linear time-varying state space model, which only necessitates a linear Kalman filter algorithm. Thus, global convergence can always be guaranteed and the nonlinearity in the model need not be considered. To further improve the robustness of the estimator with respect to the time-drifting bias in the tri-axis magnetometer, an automatic calibration method is developed. Finally, the estimation algorithm is demonstrated experimentally by using real measurement data collected from a 9-axis motion tracking sensors InvenSenses MPU9250. A comparison with the angular data measured from the encoders mounted on the joints of a 5-DOF robot is also presented. Compared with existing attitude methods reported in the literature, the primary contributions of this paper are listed below.

1. A direction cosine matrix (DCM) based attitude estimation method from the data of a 9-axis inertial measurement unit, which avoids singularities and nonlinear kinematic models, is proposed.
2. A novel algorithm to perform automatic calibration of the tri-axis magnetometer bias is developed.
3. An experimental verification of the attitude estimation results from an inexpensive smart phone grade inertial sensor using a 5-dof robot is presented.

The remainder of this paper is organized as follows. A brief review on Euler angles and their relationship with the DCM is presented in Section 2. Section 3 discusses the development of the algorithm for the estimation of the DCM in detail. The automatic estimation of bias parameters in the tri-axis magnetometer is studied in Section 4. The estimation results from the real experimental sensor data with a 5-DOF robotic arm are shown in Section 5. Section 6 contains the final conclusions.

2. Kinematic model for attitude estimation

There are three ways to parametrize the attitude of a rigid body, which are the Euler angles, direction cosine matrix (DCM) and quaternions [7,17]. The DCM approach will be proposed in this paper. However, the Euler angles and DCM are strongly related from both intuition and a mathematical perspective. The definitions of these two representations together with their basic properties will be provided in the remainder of this section.

2.1. Euler angles

The idea behind Euler rotations is to split the complete rotation of the coordinate system attached to an object into three simpler constitutive rotations, called precession, nutation, and intrinsic rotation, being each one of them an increment on one of the Euler angles. Therefore, any final orientation of an object in a 3-D space can be described using 3 rotation angles (Euler angles) in the specified orders [17,18]. Suppose $(\bar{\mathbf{I}}, \bar{\mathbf{J}}, \bar{\mathbf{K}})$ and $(\bar{\mathbf{i}}, \bar{\mathbf{j}}, \bar{\mathbf{k}})$ represent the unit vectors of the inertial coordinate frame and a body-fixed frame respectively. The rotation order is assumed to be yaw(ψ)—pitch(θ)—roll(ϕ) as shown in Fig. 1 below, where $(\bar{\mathbf{i}}_{v1}, \bar{\mathbf{j}}_{v1}, \bar{\mathbf{k}}_{v1})$ and $(\bar{\mathbf{i}}_{v2}, \bar{\mathbf{j}}_{v2}, \bar{\mathbf{k}}_{v2})$ denote the unit vectors of the intermediate coordinates after yaw and pitch rotation. The final attitude of the body-fixed frame $(\bar{\mathbf{i}}, \bar{\mathbf{j}}, \bar{\mathbf{k}})$ is uniquely determined by the Euler angles ψ, θ and ϕ [7].

The rotation matrices corresponding to the three constitutive rotations are shown below [17,19].

$$R_n^{v1} = \begin{bmatrix} c(\psi) & s(\psi) & 0 \\ -s(\psi) & c(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R_{v1}^{v2} = \begin{bmatrix} c(\theta) & 0 & -s(\theta) \\ 0 & 1 & 0 \\ s(\theta) & 0 & c(\theta) \end{bmatrix}, \quad R_{v2}^{b} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c(\phi) & s(\phi) \\ 0 & -s(\phi) & c(\phi) \end{bmatrix} \tag{1}$$

where $c(\cdot)$ and $s(\cdot)$ are the abbreviation of the $\cos(\cdot)$ and $\sin(\cdot)$ functions. The symbols $n, v1, v2, b$ denote the inertial coordinate frame, the first and second intermediate coordinate frames and the body-fixed coordinate frame respectively. Based on