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Brief paper

Containment control of continuous-time linear multi-agent systems with aperiodic sampling☆

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ABSTRACT

In this paper, the containment control problem of continuous-time linear multi-agent systems is investigated. An aperiodic sampled-data based protocol is induced by using neighboring information with uncertainly time-varying sampling intervals. By utilizing the proposed protocol and properties of Laplacian matrix, the containment control problem of continuous-time linear multi-agent systems is equivalently transformed into a stability problem of discrete-time linear systems. The stability analysis is based on the robustness of related discrete-time systems against perturbation caused by the variation of sampling intervals. By using small-gain theorem, sufficient conditions are obtained to guarantee the stability of uncertain discrete-time systems. Furthermore, two special cases are given to illustrate the method proposed in this paper. The theoretical results are verified by some simulations.

1. Introduction

Cooperative control is concerned with engineered systems that can be characterized as a collection of decision-making components with locally sensed information, limited inter-component communications, and processing capabilities, all seeking to achieve a collective objective. In recent years, due to the rapid developments of computer science and sensing/communication technologies, distributed cooperative control of multi-agent systems has made great progress. Applications of cooperative control include UAV formation control (Ryan, Zennaro, Howell, Sengupta, & Hendrick, 2004), transportation systems (Tomlin, Pappas, & Sastry, 1998), autonomous vehicle systems (Ren, Beard, & Atkins, 2007). Therefore, investigations into fundamental aspects of cooperative control of multi-agent behavior have been widely reported, e.g., consensus (Jadbabaie, Lin, & Morse, 2003; Liu, Xie, & Wang, 2010; Olfati-Saber & Murray, 2004; Ren & Beard, 2005), formation control (Ding, Yan, & Lin, 2010; Do, 2012; Lin, Francis, & Maggiore, 2005), rendezvous (Cortes, Martinez, & Bullo, 2006; Dong & Huang, 2013), flocking (Olfati-Saber, 2006; Zhang, Zhai, & Chen, 2011), and coverage control (Cortés, Martínez, Karatas, & Bullo, 2004; Zhai & Hong, 2013).

As a kind of cooperative behavior, containment control of multi-agent systems has been investigated a lot in recent years. It can be found in many application scenarios such as when a collection of autonomous robots are to secure and then remove hazardous materials. In Dimarogonas, Egerstedt, and Kyriakopoulos (2006), containment control of multiple unicycle agents was a combination of formation and agreement problems. A hybrid control scheme based on stop–go rules was proposed in Ji, Ferrari-Trecate, Egerstedt, and Buffa (2008) to guarantee that the followers remain in the convex polytope spanned by the leader-agents during their transportation. In Notarstefano, Egerstedt, and Haque (2011), it was shown that containment could be achieved as long as the time-varying undirected communication graph among all agents was jointly connected. The authors of Cao, Stuart, Ren, and Meng (2010), Lou and Hong (2012) and Li, Ren, and Xu (2012) studied containment control problems of double-integrator dynamics with switching topologies, stochastic topologies, and partial information, respectively. In Mei, Ren, and Ma (2012), containment control problem for networked Lagrangian systems with multiple

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dynamic leaders in the presence of parametric uncertainties was studied. Containment control of linear multi-agent systems under general topologies was studied in Liu, Xie, and Wang (2012a). In Yoo (2013), a containment control approach for uncertain nonlinear strict-feedback systems with multiple dynamic leaders was presented.

It is worth noting that the agents need to continuously receive information from its neighbors in most existing papers. In distributed networks, transmitting full-state information may result in high communication traffic. In Galbusera, Ferrari-Trecate, and Scattolini (2013), a hybrid model predictive control scheme is proposed for containment and distributed sensing of discrete-time systems. However, in some cases, the multi-agent system itself is a continuous process, communication between adjacent vertices are only allowed at sampling instants for the sake of cost saving. Therefore, it is necessary to study sampled-data based containment control of continuous-time multi-agent systems. In Liu, Xie, and Wang (2012b), periodic sampled-data based containment control of continuous-time multi-agent systems with single-/double-integrator dynamics was studied. It is indeed reasonable to consider the periodic sampling. However, there are some applications where the periodic sampling is impossible. For examples, in networked control systems and event-trigger control systems, the sampling operation results may be aperiodic and uncertainly time-varying. In Liu, Cheng, Tan, and Hou (2014), aperiodic sampled-data based containment control for double-integrator dynamics was studied. However, the dynamics is always more complicated, agents of higher dynamical order are required.

Motivated by the above observation, we consider aperiodic sampled-data based containment control for networks of continuous-time linear multi-agent systems in this paper. The control protocol of each agent is only based on the information measured at the sampling instants from itself and its neighborhood rather than the complete continuous process. Differently from the full-state information case, here, the closed-loop system is transformed into a discrete-time system by discretizing the continuous-time system. Utilizing properties of Laplacian matrix, the containment control problem is equivalent to a stability problem. We will study stability robustness of sampled-data systems against perturbation caused by the variation of sampling intervals, based on a small-gain modeling of the perturbation. Once we fix a sampling length in the given range, we have a time-invariant discrete-time system corresponding to it. Then the systems with aperiodic sampling intervals can be considered as the uncertain systems of the time-invariant system. If there exists a quadratic discrete-time Lyapunov function which verifies stability of all such uncertain discrete-time systems corresponding to sampling intervals in the range, the exponential stability of discrete-time systems will be derived. Namely, containment control can be achieved if the sampling size and the gain parameters are chosen properly.

The outline of this paper is shown as follows: we first establish some of the basic notations. We then, in Section 2, give some preliminaries. We give the problem formulation in Section 3 and then the main results in Section 4. In Section 5, we give two special cases to illustrate the method proposed in this paper. Section 6 gives some simulation results. Finally, the conclusions are given in Section 7.

Notations: $\mathbf{0}_{m\times n}$ denotes an all-zero matrix with dimension $m\times n$. Let I_n be the $n\times n$ identity matrix and $\mathbf{1}_n$ be the n-dimensional vector with all the elements being 1. $\mathbb{R}^{n\times n}$ and $\mathbb{C}^{n\times n}$ denote the set of real and complex $n \times n$ matrices, respectively. $\Re(\lambda)$ represents the real part of a complex number λ. $\rho(A)$ represents the spectral radius of a matrix A. $\mathbb{N}$ represents the set of natural numbers.

2. Preliminaries

In this section, some basic concepts and results are introduced. For more details, please refer to Biggs (1974), Horn and Johnson (1987), Rockafellar (1970).

Let $\mathcal{G} = (\mathcal{V}(\mathcal{G}), \mathcal{E}(\mathcal{G}), \mathcal{A}(\mathcal{G}))$ be a weighted directed graph with a set of vertices $\mathcal{V}(\mathcal{G}) = \{v_1, v_2, \ldots, v_n\}$, a set of edges $\mathcal{E}(\mathcal{G}) \subseteq \mathcal{V}(\mathcal{G}) \times \mathcal{V}(\mathcal{G})$, and a weighted adjacency matrix $\mathcal{A}(\mathcal{G}) = [a_{ij}]$. An edge of $\mathcal{G}$ is denoted by $e_{ij} = (v_j, v_i)$, where v_j is called the parent vertex of v_i and v_i the child vertex of v_j. We assume that there are no self-loops, i.e., $e_{ii} \notin \mathcal{E}(\mathcal{G})$. The adjacency elements associated with the edges are positive, i.e., $e_{ij} \in \mathcal{E}(\mathcal{G}) \Leftrightarrow a_{ij} > 0$. The set of *neighbors* of vertex v_i is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V}(\mathcal{G}) : (v_j, v_i) \in \mathcal{E}(\mathcal{G}), j \neq i\}$. A directed *path* in a directed graph $\mathcal{G}$ is a sequence $v_{i_1}, v_{i_2}, \ldots, v_{i_k}$ of vertices such that for $s = 1, 2, \ldots, k-1$, $(v_{i_s}, v_{i_{s+1}}) \in \mathcal{E}(\mathcal{G})$. A *directed tree* is a directed graph, where every vertex, except one special vertex without any parent, which is called the root vertex, has exactly one parent, and the root vertex can be connected to any other vertices through paths. A *directed forest* is a directed graph consisting of one or more directed trees no two of which have a vertex in common. A *directed spanning tree* (*directed spanning forest*) is a directed tree (directed forest), which consists of all the vertices and some edges in $\mathcal{G}$.

The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{n\times n}$ of $\mathcal{G}$ is defined as: $l_{ij} = \sum_{k=1,k\neq i}^{n} a_{ik}$ for $i = j$, and $l_{ij} = -a_{ij}$ for $i \neq j$, $i, j = 1, \ldots, n$. The *in-degree* and *out-degree* of node v_i are, respectively, defined as: $\deg_{in}(v_i) = \sum_{v_j\in\mathcal{N}_i} a_{ij}$ and $\deg_{out}(v_i) = \sum_{v_j\in\mathcal{N}_i} a_{ji}$. The *degree matrix* is an $n\times n$ matrix defined as: $D = [d_{ij}]$, where $d_{ij} = \deg_{in}(v_i)$ for $i = j$, otherwise, $d_{ij} = 0$. The Laplacian matrix of $\mathcal{G}$ can be written as: $L = D - \mathcal{A}(\mathcal{G})$.

We use directed graphs to model the communication topologies among agents. Agent i is represented by vertex v_i. Edge $e_{ij} \in \mathcal{E}(\mathcal{G})$ corresponds to an available information channel from agent j to agent i. The *neighbors* of agent i are those agents whose information is received by agent i.

Definition 1 (*Cao et al., 2010*)**.** For a group of n agents, an agent is called a *leader* if the agent has no neighbor. An agent is called a *follower* if the agent has at least one neighbor.

Assume that there are m, $2 \leq m < n$, leaders and $n-m$ followers. Denote the set of leaders as $\mathcal{R}$ and the set of followers as $\mathcal{F}$. From the definition of leaders, we know that there does not exist one directed edge from a follower to a leader, and also, there does not exist one directed edge between two leaders. Therefore, by rearranging the indices of the n agents, L can be partitioned as

$$\begin{bmatrix} L_{\mathcal{F}\mathcal{F}} & L_{\mathcal{F}\mathcal{R}} \\ \mathbf{0}_{m\times(n-m)} & \mathbf{0}_{m\times m} \end{bmatrix}, \tag{1}$$

where $L_{\mathcal{F}\mathcal{F}} \in \mathbb{R}^{(n-m)\times(n-m)}$ and $L_{\mathcal{F}\mathcal{R}} \in \mathbb{R}^{(n-m)\times m}$.

The following lemma plays an important role in the containment control analysis.

Lemma 1 (*Liu et al., 2012b*)**.** *$L_{\mathcal{F}\mathcal{F}}$ is invertible if and only if the directed graph $\mathcal{G}$ has a spanning forest.*

Definition 2 (*Rockafellar, 1970*)**.** A set $H \subset \mathbb{R}^m$ is said to be convex if $(1-\gamma)x + \gamma y \in H$ whenever $x \in H$, $y \in H$ and $0 \leq \gamma \leq 1$. The convex hull of a finite set of points $x_1, x_2, \ldots, x_n \in \mathbb{R}^m$ is denoted by $\mathrm{co}\{x_1, x_2, \ldots, x_n\} = \{\sum_{i=1}^{n} \alpha_i x_i | \alpha_i \in \mathbb{R}, \alpha_i \geq 0, \sum_{i=1}^{n} \alpha_i = 1\}$.