Mechanical Systems and Signal Processing 108 (2018) 115–125

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

System identification of energy dissipation in a mechanical model undergoing high velocities: An indirect use of perpetual points

P. Brzeski [a,*], L.N. Virgin [b]

[a] *Division of Dynamics, Lodz University of Technology, 90-924 Lodz, Poland*
[b] *Department of Mechanical Engineering and Materials Science, Duke University, Durham, NC 27708-3000, United States*

ARTICLE INFO

Article history:
Received 24 October 2017
Received in revised form 3 February 2018
Accepted 5 February 2018

Keywords:
Perpetual points
Dissipation models
Mechanical systems
Experimental investigation
Pendulum

ABSTRACT

Energy dissipation is often the most challenging component of system identification in the modeling of dynamical behavior in mechanical systems. Even for a relatively simple single-degree-of-freedom system such as the rigid-arm pendulum, it can be difficult to choose the form of the best damping model, as well as the subsequent challenge of estimating the appropriate parameters, especially for a model that accurately captures the nature of energy dissipation over a wide range of operational conditions. This paper specifically focuses on a mechanical system in which subtle changes can be made to the system with a view to isolating and modeling energy dissipation. The approach described in this paper was developed as a by-product of experimentally investigating perpetual points. It is shown that certain features of high-velocity, *spinning* motion lends itself to greater fidelity in the data-fitting process and thus added confidence in choosing the most accurate energy dissipation model with the most appropriate parameters.

1. Introduction

In a previous study by the same authors [5], an investigation was designed to identify and locate perpetual points (PPs) in an experimental mechanical system, i.e., points for which accelerations of the system becomes zero while velocities remain nonzero [22]. PPs gain scientific interest including some interesting applications such as confirming energy conservation of dynamical systems [22,12] or to locate hidden and rare attractors [7,18]. In the process of modeling the system – a quite simple, tilted, rigid-arm pendulum undergoing relatively high-velocity spinning, the major components of the mechanical system (stiffness, inertia) are measured and modeled with relative ease. However, in a situation not uncommon in experimental mechanics, the characterization of damping naturally became a focus of attention. It is often a challenging task to develop a reliable energy dissipation model and optimize its parameters [11,15,21,17,13].

Most often, the starting point for the modeling of energy dissipation is based on linear viscous damping, i.e., a force that is directly proportional to velocity. In addition to some physical justification, this approach has been popular due to its non-violation of linearity and the utility of the principle of superposition for example. Moreover, viscous damping forms the basis of relatively simple measurement techniques including the logarithmic decrement and half-power methods [1,2]. Rubbing between dry surfaces is typically modeled by Coulomb damping, and despite a lack of consensus about the subtle physical

* Corresponding author.
E-mail address: piotr.brzeski@p.lodz.pl (P. Brzeski).

https://doi.org/10.1016/j.ymssp.2018.02.010
0888-3270/

interactions at very small scales (e.g., micro-slip), it is an approach that has been popularly applied in many applications. At higher velocities, Coulomb damping plays a much less important role, but it is well-established that due to interaction with a fluid drag can become important [13], i.e., a force assumed to be proportional to velocity squared. And then it seemed appropriate to include the possibility of cubic viscous damping, which is clearly likely the dominant form of energy dissipation at very high velocities. However, it is seldom clear whether these forms act in isolation or together [19,14,8], whether there might be damping related to the cube of the velocity say [9], or even amplitude-dependent damping [6]. However, a proper identification of internal damping is crucial for simulation accuracy for many different phenomena such as synchronization [4] or modeling of mechanical devices [3].

In the process of identifying and characterizing PPs, it was noticed that relatively subtle differences in energy dissipation models affect their location significantly. Thus, the location of perpetual points seems to provide an opportunity: exploiting this sensitive relation with damping (both in terms of the model and the parameter value(s) chosen) as an identification procedure. Hence, comparing experimental and numerical data (based on the various assumed damping models) we are able to find appropriate damping model and tune it. Basing on this observation we elaborated model identification procedure in which we utilize large number of trials with high-velocity spinning of the experimental pendulum, with the angle of tilt used as a convenient control parameter. The pendulum was tilted (out of plane) at an angle close to horizontal in order to reduce the natural frequency, and transient behavior was initiated by imparting a sudden, high-magnitude impulse directly to the system. The resulting angular motion was then measured to high accuracy with a multi-turn potentiometer attached to the pivot, thus generating free-decay time series. This data was then compared with various numerical models, in which the damping took on a number of mathematical forms. Because the presented approach is based on PPs that are located at high velocities, the proposed energy dissipation models should work well especially for velocities of magnitude comparable to the velocities associated with the PP. The following sections describe details on how this process was implemented, resulting in some encouraging results pertaining to the modeling and parameter estimation of energy dissipation.

2. Methodology

A number of methods have been developed to extract the viscous damping coefficient and dry friction force. Feeny and Liang [8,14] propose a method that uses time traces of free small amplitude oscillations. In [16] the authors apply energy balance method to calculate coefficients for the forced system. However, most of the methods are reliable for the analysis of harmonic oscillations of linear systems [13,10] and there are a lack of efficient methods that enable validation of energy dissipation model for high velocities [20,1,2]. Since PPs in mechanical systems naturally occur at large velocities and strongly depend on the dissipation we describe an attempt to utilize experimentally obtained PPs to assess the suitability of energy dissipation models.

We propose the following procedure to evaluate and assess various energy dissipation models:

1. We select one parameter of the system that we can easily change but whose changes does not affect energy dissipation significantly. In this case we choose the angle of tilt.
2. We derive the analytical formula that describes how the change in this parameter value influences the location of PPs.
3. For a number of different values of the selected parameter we experimentally obtain PPs. The minimum number of the analyzed parameter values is three, but the reliability of the assessment grows with the number of samples.
4. We then adjust the parameters values to obtain a better fit to the experimental data.
5. Finally, we compare the experimentally obtained PPs with the optimized model to asses the accuracy of the approach.

3. Model of the system

In the paper we experimentally investigate the dynamics of a tilted physical pendulum with the specific configuration as presented in Fig. 1. The system was specifically designed in order to facilitate locating PPs experimentally. As part of this process, a plastic strip was added in order to increase the drag, and the axis of rotation was tilted. The pendulum was mounted directly to the rotary motion sensor (PASCO CI-6538) which enabled precise measurement of the pendulum position (angle) and serves as a pin-joint. It is not atypical for such a pivot point to exhibit a degree of dry friction. In terms of overall magnitude the energy dissipation was considered to be extremely light, with the pendulum able to undergo dozens of rotations after a moderately large kick before the system eventually came back to rest.

In Fig. 1 (b) we show the schematic model of the pendulum rod with its parameters. The pendulum has the mass m, the moment of inertia I and the centre of gravity located at the distance l from its pin joint. The state of the system is described by the angular displacement of the pendulum φ. The axis of rotation is tilted from vertical line by the angle α.

The governing equation of motion of the system shown in Fig. 1 is described by the following second order ODE:

$$I\ddot{\varphi} + mgl\sin(\alpha)\sin(\varphi) + T(\dot{\varphi}) = 0, \tag{1}$$

in which primary subsequent interest will be focused on the specific form adopted for $T(\dot{\varphi})$. The physical parameters have the following values: $I = 5.2502 \cdot 10^{-5}$ [kg m^2], $m = 0.017$ [kg], $l = 5.0 \cdot 10^{-2}$ [m], $g = 9.81$ [m/s^2]. The angle of inclination (tilt)