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Natural frequency assignment for mass-chain systems with inerters

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ABSTRACT

This paper studies the problem of natural frequency assignment for mass-chain systems with inerters. This is the problem to determine whether an arbitrary set of positive numbers may be assigned as the natural frequencies of a chain of n masses in which each element has fixed mass and is connected to its neighbour by a parallel combination of a spring and inerter. It is proved that mass-chain systems with inerters may have multiple natural frequencies, which is different from conventional mass-chain systems (without inerters) whose natural frequencies are always simple. It is shown that arbitrary assignment of natural frequencies including multiplicities is not possible with the choice of n inerters and n springs. In particular, it is shown that an eigenvalue of multiplicity m may occur only if $n \geqslant 2m - 1$. However, it is proved that $n - 1$ inerters and n springs are necessary and sufficient to freely assign an arbitrary set of *distinct* positive numbers as the natural frequencies of an n-degree-of-freedom mass-chain system.

1. Introduction

The inerter is a two-terminal mechanical device with the property that the applied force at the terminals is proportional to the relative acceleration between them, with the constant of proportionality termed the inertance [1]. One of the principal motivations for the inerter is to achieve a complete correspondence between mechanical and electrical network elements [1,2]. An important property of the inerter is that a large inertance can be obtained by devices of relatively small physical mass. As a result, the inerter can used in the control of mechanical systems without adding to the overall mass of the system. Up to now, the inerter has been applied to various systems such as vehicle suspensions [3–6], train suspensions [7,8], buildings [9,10], dynamic vibration absorbers [11–13], vibration isolators [14], landing gears [15], passive mechanical networks [16–18], etc. The inerter has also been successfully used in Formula One racing since 2005 [2].

In this paper, the natural frequency assignment problem for mass-chain systems with inerters is studied, where the adjacent masses are connected by a parallel combination of a spring and an inerter. Mass-chain systems are common mechanical systems, which can be used to describe a variety of mechanical systems such as multi-storey buildings [10,19], dynamic vibration absorbers [11,20], vehicle models [3–6], finite-element models of continuum mechanical systems [21], and so on. Natural frequency is one of the most important inherent properties for mechanical vibration systems, similar to the poles of control systems, determining the dynamic behaviours of mechanical vibration systems. If the excitation frequency is close

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to one of the natural frequencies, resonance may occur. In practice, it is always desirable to assign the natural frequencies of a vibration system to some specified values or regions such that resonance can be induced [22] or avoided [23]. Therefore, the natural frequency assignment problem for mechanical systems is of practical importance, and it has received much attention [24–27].

The conventional passive way of achieving natural frequency assignment is by carefully choosing the masses and spring stiffnesses, and it is well known that increasing masses and spring stiffnesses can reduce and increase natural frequencies, respectively. For the mass-chain system where the adjacent masses are connected only by a spring, it has been demonstrated that all natural frequencies are distinct [21], and any n arbitrarily given positive distinct numbers can always be realized as the natural frequencies of an n-degree-of-freedom (DOF) system by appropriate choice of the n masses and n spring constants [28]. However, in practice, the masses are normally given with fixed parameters. In such cases the spring stiffnesses alone offer a more limited freedom to adjust the natural frequencies. In [29], it has been demonstrated that if inerters are introduced in parallel to the springs then increasing their inertance can effectively reduce a mechanical systems' natural frequencies. Thus inerters offer a new design possibility in cases where it is not feasible to adjust the masses in a system.

By considering inertances and spring stiffnesses as the design parameters for mass-chain systems with the masses fixed, a fundamental question arises: whether it is possible to realize any arbitrarily given real positive numbers as the natural frequencies of mass-chain systems, and if so, what is the minimal number of inerters required to achieve this. This question will be addressed in this paper by formulating the problem as a direct problem and an inverse problem. The direct problem is an analysis problem, where the multiplicity of a mass-chain system's natural frequencies will be analysed. A difference between mass-chain systems with and without inerters will be demonstrated in that it is possible for mass-chain systems with inerters to have multiple natural frequencies, while the natural frequencies of mass-chain systems without inerters are always simple and distinct. The case of multiple eigenvalues will be studied using a recurrence relation that defines the eigenvalues. It will be shown that there are restrictions on the multiplicities which may occur. In particular it will be shown that an eigenvalue of multiplicity m may occur only if $n \geqslant 2m-1$, and necessary and sufficient conditions will be derived for the case of $n = 2m-1$. In contrast, if all the given n positive numbers are distinct, from a synthesis point of view and by using an inverse eigenvalue problem formulation, it will be proved that it is necessary and sufficient to use $n-1$ inerters and n springs to freely assign any arbitrarily given numbers as the natural frequencies of an n-degree-of-freedom mass-chain system.

The rest of this paper is organized as follows. Section 2 formulates the natural frequency assignment problem as an eigenvalue assignment problem, and its relation with pole placement problem is introduced. In Section 3, the direct problem of analyzing the multiplicity of natural frequencies is investigated. Section 4 addresses the natural frequency assignment problem where all the given numbers are distinct. Conclusions are drawn in Section 5.

2. Problem formulation

The mass-chain system shown in Fig. 1 is considered. The free vibration equation is

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = 0,$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$,

$$\mathbf{M} = \mathbf{M}_0 + \mathbf{B}, \tag{1}$$

$$\mathbf{M}_0 = \mathrm{diag}\{m_1, m_2, \ldots, m_n\}, \tag{2}$$

$$\mathbf{B} = \begin{bmatrix} b_1+b_2 & -b_2 & & & \\ -b_2 & b_2+b_3 & -b_3 & & \\ & \ddots & \ddots & \ddots & \\ & & -b_{n-1} & b_{n-1}+b_n & -b_n \\ & & & -b_n & b_n \end{bmatrix}, \tag{3}$$

$$\mathbf{K} = \begin{bmatrix} k_1+k_2 & -k_2 & & & \\ -k_2 & k_2+k_3 & -k_3 & & \\ & \ddots & \ddots & \ddots & \\ & & -k_{n-1} & k_{n-1}+k_n & -k_n \\ & & & -k_n & k_n \end{bmatrix}. \tag{4}$$

The natural frequencies of the mass-chain system are determined by the square roots of the eigenvalues of the matrix pencil $(\mathbf{K} - \lambda\mathbf{M})$, the eigenvalues being the roots of the following characteristic equation

$$\det(\mathbf{K} - \lambda\mathbf{M}) = 0. \tag{5}$$

In the following we will use the terms "eigenvalues" and the "natural frequencies" interchangeably. The following problem is studied in this paper.