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Control of a class of second-order linear vibrating systems with time-delay: Smith predictor approach

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A B S T R A C T

This paper presents an innovative approach to the design of feedback control in second-order symmetric linear systems under long input time-delay. Recent works have presented design techniques for second order-systems. However, most of them carry out an *a posteriori* analysis to check if the target eigenvalues are correctly assigned. An inaccurate eigenvalue assignment can occur due to infinite dimensional nature of the characteristic polynomial for systems with time-delay, and, eventually, the target eigenvalues can be assigned as secondary ones, with unstable, dangerous primary eigenvalues entering the picture. The proposal uses a Smith Predictor based approach to compensate the time-delay by providing a nominal characteristic polynomial without delay. The prediction is obtained by employing the versatile receptance approach in the conception of the predictor, and thus the design is entirely made at the frequency domain. The proposal uses a filtered prediction error which can be applied to attenuate the undesired effect of the poorly damped eigenvalues and ensure the internal stability of the marginally stable case. Some numerical experiments are given to illustrate the effectiveness of the proposed approach.

1. Introduction

The control of second-order linear system has gained more interest in recent years. Second-order models are a natural way to describe mechanical vibrations, oscillations in electrical networks, vibroacoustic phenomena, among others [1–4]. One of the central problems in the design of feedback control for second-order linear systems is the well-known eigenstructure assignment. The problem is solved computing feedback matrices for state, derivative or output feedback such that the eigenvalues and eigenvectors lie in prescribed positions [5–11]. Since that the eigenvalue problem in second-order system is of quadratic type – QEP: Quadratic Eigenvalue Problem [12], there are works in the literature that explore some properties of this kind of problem. One of the main advantages is the manipulation of the model matrices in a given dimension, without the necessity of using a doubled dimension, equivalent first-order state space representation. Moreover, the symmetry of the system matrices, a somewhat often property in second-order models, is lost when one considers it first-order equivalent model [13]. Another powerful approach to solve the eigenstructure assignment problem is the receptance approach [14,15]. Full or partial pole assignment is possible with this method with only the known of the receptance matrix of the system, and it can be obtained using data from frequency response experiments. Other interesting possibilities for design

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using receptances is the inclusion of robustness as a performance criterion [16,17]. The design of feedback matrices turns a little bit more involved for systems with internal time-delays.

Internal time-delays are present in several practical applications, due to a series of reasons, as the physical separation between sensors locations and the ideal measurement points, communication delay in networked control systems or models of phenomena with distributed parameters [18]. The receptance approach and others have been addressed for systems with internal time-delays [19–23]. Unfortunately, closed-loop stability property may be lost in the presence of time-delay, due to the infinite dimensional nature of the closed-loop characteristic polynomial. Some expected eigenvalues can be erroneously placed as secondary ones, and unstable, dangerous primary eigenvalues enter the picture. Thus, all these works conduct *a posteriori* analysis of the closed-loop eigenvalues to check its stability.

Time-delay compensation concept has already been proposed to control second-order systems [24]. In [24], the SP approach is employed in comparison with other well-known techniques as LQR + Padé approximation, but the system is reformulated as a first-order realization, and hence a doubled dimensional model is used. The approach given in [25] apply SP to time-delay compensation in vibrating systems but only consider the SISO case. Other interesting papers as [26,27] dealt with time-delay compensation differently from SP, but again the first-order equivalent model is issued. The results of [27] use pole placement approach, but no guarantee of stability is provided. Besides these drawbacks, the approaches in [26,27] are recommended for systems with small values of time-delay. In none of this works the repectance was directly employed for prediction purposes. This is an essential drawback for some applications since receptance can be directly obtained by using experimental apparatus with sensors/actuators measurements. Moreover, the standard Smith predictor cannot be used to control marginally stable systems in Lyapunov sense because closed-loop system is not internally stable [28].

In the present work, we propose the use of a modified Smith predictor based approach [18,29–31,28,32] to restore the closed-loop eigenvalues as the same in the design of the system in the absence of time-delay. Again, the receptance matrix is demonstrated to play a relevant role in the approach, leading to a virtually whole design in the frequency domain. This modified time-delay compensator has important contributions in the context of second-order systems: (i) marginally stable systems can be controlled since internal stability is ensured, (ii) undesired poorly damped modes can be attenuated due to the prediction error filter, (iii) predictions can be computed by using the receptance, and (iv) the augmented first-order description is not used.

The next sections are organized as the preliminaries and a brief statement of the problem, followed by the receptance-based time-delay compensation description and some numerical experiments with comparisons. In the concluding remarks section, an overview is made, and future research possibilities are stated.

2. Preliminaries and statement of the problem

We consider second-order linear systems in the form:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t), \tag{1}$$

where $\mathbf{M}, \mathbf{C}$ and $\mathbf{K}$ are, respectively, the mass, damping and stiffness matrices, each of order n, $\mathbf{x}(t)$ is the displacement vector and $\mathbf{f}(t)$ is a external vector force. Moreover, the systems studied are such that $\mathbf{M} = \mathbf{M}^T \succ 0, \mathbf{C} = \mathbf{C}^T \succeq 0$ and $\mathbf{K} = \mathbf{K}^T \succ 0$. Let $\mathbf{B} \in \mathbb{R}^{n\times m}$ be an influence matrix that represents the actuator configuration. The later is composed of two components as:

$$\mathbf{f}(t) = \mathbf{B}\mathbf{u}(t-\tau) + \mathbf{d}(t) \tag{2}$$

where $\mathbf{u}(t) \in \mathbb{R}^m$ is the control vector and $\mathbf{d}(t) \in \mathbb{R}^n$ is a bounded external disturbance, and $\tau > 0$ is the input time-delay. Taking the Laplace Transform of (1), the displacement vector can be expressed in explicit form in the frequency domain:

$$\mathbf{X}(s) = e^{-\tau s}\mathbf{H}(s)\mathbf{B}\mathbf{U}(s) + \mathbf{H}(s)[(\mathbf{M}s + \mathbf{C})\mathbf{x}(0) + \mathbf{M}s\dot{\mathbf{x}}(0)] + \mathbf{H}(s)\mathbf{D}(s) \tag{3}$$

where $\mathbf{H}(s) = (\mathbf{M}s^2 + \mathbf{C}s + \mathbf{K})^{-1}$ is known as open-loop system receptance, or simply receptance. The concept of receptance plays a central role in feedback control design of a second-order system, as can be seen in the seminal work [14] and several subsequent ones – see [16] and references therein, and also [23,33,17]. The receptance approach takes advantage of the use of experimental measurements which can be used to obtain its estimation, without the necessity of the knowledge of the system matrices in some cases, or the improvement when applied in mixed methods involving the use of receptances and system matrices.

The central problem of the present work can be stated as follows: given the system (1), one must design a linear feedback controller that match as much as possible the control objective and simultaneously compensate possible long and constant input time-delays.

The majority of methods to eigenvalue or eigenstructure assignment by linear feedback – state, output or derivative, compute the feedback matrices that are supposed to be capable of to provide the target allocation. However, the target location is part of an infinite set of eigenvalues, due to the transcendental nature of the closed-loop characteristic polynomial. Then, primary unstable eigenvalues can be the true location of the closed loop eigenvalues. In the next section, we propose a solution by using versions of SP, and the possible drawbacks that lead to a necessity of using filtered versions of that.