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Comparison of harmonic balance and multi-scale method in characterizing the response of monostable energy harvesters ☆

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ABSTRACT

Harmonic balance and multi-scale methods have been widely applied to investigate the nonlinear dynamics and performance enhancement methods of nonlinear vibration energy harvesters. However, there are rare works considering the relative accuracy of the two methods in characterizing the response of nonlinear harvesters. Therefore, new insights into the analytical accuracy of harmonic balance and multi-scale methods are provided based on a monostable energy harvester in this paper. Frequency response functions of displacement and voltage for a monostable harvester are derived according to harmonic balance expansion and multi-scale method respectively. Monostable system parameters identified from experimental measurements are obtained for numerical investigation of the theoretical solution from harmonic balance and multi-scale methods. Results demonstrate that harmonic balance method is more accurate to characterize the frequency response under different acceleration levels, as well as the amplitude response for various excitation frequencies. While for multi-scale method, it is of great importance to choose proper keeping parameter to describe the nearness of excitation frequency to the resonance frequency.

1. Introduction

Recently, vibration energy harvesting techniques have received considerable attention due to its promising ability to convert ambient vibration energy to useful electrical energy for the supply of electricity for low-power consumption devices such as sensors, wireless transceivers and portable electronic devices [1–3]. A variety of energy harvesters based on piezoelectric [4] and electromagnetic [5] mechanisms have been proposed to harvest energy from environmental vibrations, among which piezoelectric energy harvesting has been viewed as a promising method due to its high energy density and easily miniaturized fabrication. However, traditional linear harvester only performs well near the resonance frequency and slight change in the frequency will decrease its efficiency obviously. Therefore, the theoretical analysis and experimental validation of the frequency bandwidth and performance enhancement of harvesters as a result of introducing nonlinear phenomenon has received significant interest. Particularly, monostable [6], bistable [7,8], and tristable [9] oscillators with different types of potential functions attracted lots of attentions.

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To characterize the response of these nonlinear energy harvesters, harmonic balance method (HBM) and multi-scale methods (MSM) are widely applied to theoretically analyze the results. The basic idea of harmonic balance method is expanding the excitation and solution of a system into the forms of Fourier series and then balancing the terms multiplied by same harmonics. While for multi-scale method, it considers the solution of a system in different time scales and then obtain the system response by eliminating the secular terms. Zhou et al. [10,11] theoretically analyzed the influence of system parameters on the responses of monostable and tristable harvesters by using the harmonic balance method, concluding that high-energy interwell oscillations in the multi-solution ranges of the harvesters is feasible for improving energy harvesting from low-level ambient excitations. Stanton et al. [12] applied the method of harmonic balance to analytically predict the existence, stability, and influence of parameter variations on the intra-well and inter-well oscillations of bistable piezoelectric inertial generator. Moreover, Liu et al. [13] investigated the inter-well motion of buckled-spring-mass generator by applying the harmonic balance method with special doubled frequency voltage response. Harne and Wang [14] analyzed the superharmonic effects in bistable energy harvesting through the method of harmonic balance having fundamental and superharmonic components, and demonstrated that superharmonic effects could improve the performance of the bistable harvester in a certain frequency range. Furthermore, Mann et al. [15] proposed a novel electromagnetic energy harvesting device and investigated the response of the monostable harvester by multi-scale method. Daqaq et al. [16] established a model of a parametrically excited cantilever-type harvester and the multi-scale method was applied to obtain approximate analytical expressions describing the beam response. Chen and Jiang [17] derived the amplitude–frequency response relationships of the displacement and the power in the first primary resonances with the two-to-one internal resonance through the method of multiple scales and numerical results demonstrate the effectiveness of the theoretical results. Panyam et al. [18] utilized the multi-scale method to construct analytical solutions describing the amplitude and stability of the intra- and inter-well dynamics of the harvester, based on which the influence of time constant ratio, electromechanical coupling and the shape of the potential function on the effective frequency bandwidth was analyzed. Although harmonic balance and multi-scale methods have been widely used to investigate the response of nonlinear energy harvesting system, the relative accuracy of the two methods in characterizing the response of nonlinear harvester remains rather limited results.

Therefore, this paper investigates the response of a monostable energy harvester by harmonic balance and multi-scale methods to compare the relative accuracy of the two methods in characterizing the response of nonlinear harvesters. Frequency response functions of displacement and voltage for a monostable harvester are derived according to harmonic balance expansion and multi-scale method respectively. Moreover, two different ways of describing the nearness of excitation frequency to the resonance frequency are considered in multi-scale method. Identified parameters from experiments are adopted to do simulation to compare with the theoretical results and results show that harmonic balance method is more accuracy to characterize the frequency response and amplitude response under different excitations. For multi-scale method, it is of great significance to choose proper keeping parameter to characterizing the nearness of excitation frequency to the resonance frequency.

2. Model of the monostable piezoelectric energy harvester

The schematic diagram of the monostable energy harvester is illustrated in Fig. 1. The configuration is composed of a stainless steel substrate, two symmetric PZT-51 piezoelectric layers at the root, tip magnet attachments and external magnets. The tip magnets can be viewed as proof concentrated mass to change the fundamental frequency. Due to the interaction with external magnet, the magnetic force actuating on the cantilever could be changed by altering parameters d, h, and ϑ.

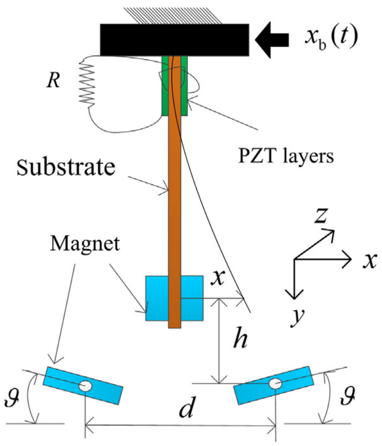

Fig. 1. Schematic diagram of the monostable energy harvester.