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Brief paper

# Functional unknown input reconstruction of descriptor systems: Application to fault detection ☆

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ABSTRACT

The problem of finding conditions for the reconstruction of a linear combination of the vector of unknown inputs is tackled. Appropriate definitions are given in order to address formally the analysis. Thus, necessary and sufficient conditions are obtained for two cases, when the reconstruction may be achieved in finite time and when it is possible to do it asymptotically. The results are applied for the fault detection, with the respective deduction of analogous conditions.



## 1. Introduction

The fault detection problem of dynamic systems has been largely studied in the last decades. One of the most known approaches used to tackle this problem has been that of using residuals, which main idea is to compare the value of the actual system output with an expected value of the output for which it is assumed that the system is free from faults. For such an approach two seminal works for linear and nonlinear systems are found in De Persis and Isidori (2001) and Massoumnia, Verghese, and Willsky (1989), respectively. In those papers, conditions are given under which the fault detection can be achieved. Other approach consists in using unknown input observers, which allows for the estimation of the faults also, see for instance Hammouri and Tmarc (2010), Hou and Patton (1998), Hui and Zak (2005) and Xiong and Saif (2003). In Bejarano (2011) and Bejarano, Figueroa, Pacheco, and Rubio (2012) one can find a procedure to reconstruct the fault signals of linear systems that may be affected by disturbances also. There, the fault reconstruction is carried out directly without the need of any observer. For the case of descriptor systems the fault detection problem has been studied in less proportion. In Gaoa and Ding (2007), the fault actuator estimation is tackled for a sort of nonlinear systems. For linear systems, in Duan, Howe, and Patton (2002), Hamdi, Rodrigues, Mechmeche, Theilliol, and Braiek (2012) and Koenig (2005) the fault detection is carried out by means of unknown input observers. The purpose of this work is to find conditions allowing for the reconstruction of the faults considering that the system is affected by disturbances (uncertainties or exogenous perturbations) also. Thus, we assume that faults may appear in the following descriptor system

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + B_1 f(t) + B_2 d(t) \\ y(t) &= Cx(t) + D_1 f(t) + D_2 d(t) \end{aligned} \tag{1}$$

there, $f(t)$ is the vector of faults and $d(t)$ is the vector of disturbances affecting the system, the matrix $E$ is **not invertible** (and not necessarily a square matrix). The procedure used to analyze the conditions for the fault reconstruction is as follows. We notice that $f$ and $d$ can be put together in a single vector, let us say $\varphi(t)$. Then $f(t)$ is equal to $L\varphi(t)$ for some matrix $L$. Hence, in the way to give answer to the main question of this work, necessary and sufficient conditions are obtained for that $L\varphi(t)$ be reconstructible in finite time and for that $L\varphi(t)$ be reconstructible asymptotically.

The structure of the paper is as follows. In Section 2 definitions of finite time and asymptotic reconstructibility are formulated. The main results regarding the reconstructibility concepts are presented in Section 3. The general results are applied to the fault reconstruction problem in Section 4, which is separated into three subsections. The steps to be followed for checking the reconstructibility of the faults are given in 4.1. Explicit procedures to reconstruct the faults are given in 4.2, for the case when the fault reconstruction is possible in finite time, and in 4.3, for the

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case when the reconstruction can be achieved asymptotically. Throughout the paper, we will use $\mathbb{R}$, $\mathbb{C}$, and $\mathbb{C}^-$ to denote the real field, the complex field, and the set of complex numbers with negative real part, respectively.

## 2. Reconstructibility definitions

Let us consider the sort of systems governed by the following equations

$$E\dot{x}(t) = Ax(t) + B\varphi(t) \tag{2a}$$
$$y(t) = Cx(t) + D\varphi(t) \tag{2b}$$
$$z(t) = L\varphi(t) \tag{2c}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^p$ is the system output, $\varphi(t) \in \mathbb{R}^m$ is a vector of unknown inputs, which are assumed to be piece-wise continuous. The vector $z(t) \in \mathbb{R}^{\bar{m}}$ represents a linear functional of $\varphi(t)$. All matrices are assumed to be constant, $E \in \mathbb{R}^{n_1\times n}$, $A \in \mathbb{R}^{n_1\times n}$, $B \in \mathbb{R}^{n_1\times m}$, $C \in \mathbb{R}^{p\times n}$, $D \in \mathbb{R}^{p\times m}$, $L \in \mathbb{R}^{\bar{m}\times m}$. In particular the matrix $E$ **is assumed to have rank less than** $n$. Without loss of generality, it is assumed that $\operatorname{rank} L = \bar{m}$.

It is assumed that $Ex(0^-)$ and $\varphi(t)$ are such that a solution of the state equation (2a) exists ($x(0^-) := \lim_{t\to 0^-} x(t)$) (see, e.g. Hou & Müller, 1999). In fact such a solution, in general, **is not unique.**[1] Nevertheless, it is assumed that every solution of $x(t)$ is piecewise differentiable.

The aim is to find conditions under which the reconstruction of $z(t)$ can be carried out using the knowledge of the system output. One possibility is to do the reconstruction of $z(t)$ in finite time, and other possibility is to do it asymptotically. Hence, to deal with the problem under study, the following definitions are given.

**Definition 1.** $z(t)$ of system (2) is finite time reconstructible (**FTR**) if the identity $y(t) = 0$ for all $t \geq 0$ implies $z(t) = 0$ for all $t \geq 0$.

**Definition 2.** $z(t)$ of system (2) is asymptotically reconstructible (**AR**) if the identity $y(t) = 0$ for all $t \geq 0$ implies $z(t) \to 0$ as $t \to \infty$.

Let $\bar{L}$ be a matrix so that $\bar{L} \in \mathbb{R}^{m\times(m-\bar{m})}$, $\operatorname{rank}\bar{L} = m - \bar{m}$, and $L\bar{L} = 0$. Matrices $L$ and $\bar{L}$ are concatenated (in case $\bar{m} < m$) to form the nonsingular matrix (and its inverse)

$$\Psi = \begin{pmatrix} L \\ \bar{L}^+ \end{pmatrix}, \qquad \Psi^{-1} = \begin{pmatrix} L^+ & \bar{L} \end{pmatrix}$$

where $\quad L^+ = L^T\left(LL^T\right)^{-1} \quad$ and $\quad \bar{L}^+ = \left(\bar{L}^T\bar{L}\right)^{-1}\bar{L}^T.$

Thus, we see that $\varphi(t) = L^+z(t) + \bar{L}\bar{z}(t)$, where $\bar{z}(t) = \bar{L}^+\varphi(t)$. In the case that $\bar{m} = m$ then, by definition, $\Psi = L$ and $\bar{L} = 0$. Now, let $\mathscr{V}^*$ be the largest subspace that satisfies the following inclusion (see, e.g. Geerts, 1993)

$$\begin{pmatrix} A \\ C \end{pmatrix}\mathscr{V} \subset (E\mathscr{V} \times 0) + \operatorname{im}\begin{pmatrix} B \\ D \end{pmatrix}. \tag{3}$$

Likewise $\bar{\mathscr{V}}^*$ is the largest subspace satisfying the inclusion

$$\begin{pmatrix} A \\ C \end{pmatrix}\bar{\mathscr{V}} \subset \left(E\bar{\mathscr{V}} \times 0\right) + \operatorname{im}\begin{pmatrix} B\bar{L} \\ D\bar{L} \end{pmatrix}. \tag{4}$$

By comparing (3) and (4), it is clearly seen that

$$\bar{\mathscr{V}}^* \subset \mathscr{V}^*. \tag{5}$$

[1] For the case when $n_1 = n$, a condition guaranteeing a unique solution of (2a) is that there exists $s_0 \in \mathbb{C}$ such that $\det(s_0E - A) \neq 0$. Such a condition is not assumed in this manuscript.

## 3. Functional unknown input reconstructibility

Let us choose a full column rank matrix $V$ so that $\operatorname{im} V = \mathscr{V}^*$. Let $M_*$ be selected as a full row rank matrix so that $M_*V = 0$ (i.e. $\ker M_* = \mathscr{V}^*$). Likewise $\bar{V}$ and $\bar{M}_*$ are calculated, except that $\bar{\mathscr{V}}^*$ has to be used instead of $\mathscr{V}^*$. An easy algorithm to calculate $M_*$ and $V$ is given in Appendix A. In view of (5), $V$ can be selected so that $V = \begin{pmatrix} \tilde{V} & \bar{V} \end{pmatrix}$. By (3), there exists a pair of matrices $(F, Q)$ such that

$$\begin{aligned} AV + BF &= EVQ \\ CV + DF &= 0 \end{aligned} \tag{6}$$

and by (4) there exists a pair of matrices $\left(\bar{F}, \bar{Q}\right)$ such that

$$\begin{aligned} A\bar{V} + B\bar{L}\bar{F} &= E\bar{V}\bar{Q} \\ C\bar{V} + D\bar{L}\bar{F} &= 0. \end{aligned} \tag{7}$$

Thus, since $V = \begin{pmatrix} \tilde{V} & \bar{V} \end{pmatrix}$, $F$ can be chosen to have the form $F = \begin{pmatrix} \tilde{F} & \bar{L}\bar{F} \end{pmatrix}$, for some matrix $\tilde{F}$. As for $Q$, we have that it has the form $Q = \begin{pmatrix} Q_1 & 0 \\ Q_2 & \bar{Q} \end{pmatrix}$. Now, let us define $M_*^+ = M_*^T\left(M_*M_*^T\right)^{-1}$ and $V^+ = \left(V^TV\right)^{-1}V^T$. Then, the following change of coordinates is defined, $w_1 \triangleq M_*x$ and $w_2 \triangleq V^+x$. Defining the vector $w$ as $w = \begin{pmatrix} w_1^T & w_2^T \end{pmatrix}^T$, we obtain, in case $\mathscr{V}^* \neq \bar{\mathscr{V}}^*$,

$$EM_*^+\dot{w}_1 + EV\dot{w}_2 = A_eM_*^+w_1 + EVQw_2 + B(\varphi - Fw_2) \tag{8a}$$
$$y = C_eM_*^+w_1 + D(\varphi - Fw_2) \tag{8b}$$

where $A_e \triangleq A + BFV^+$ and $C_e \triangleq C + DFV^+$. Indeed, (8) is easily obtained by taking into account (6) and the fact that $V^+V = I$. In case $\mathscr{V}^* = \bar{\mathscr{V}}^*$, we obtain

$$EM_*^+\dot{w}_1 + EV\dot{w}_2 = A_eM_*^+w_1 + EVQw_2 + B\left(\varphi - \bar{L}\bar{F}w_2\right) \tag{9a}$$
$$y = C_eM_*^+w_1 + D\left(\varphi - \bar{L}\bar{F}w_2\right) \tag{9b}$$

there, $A_e \triangleq A + B\bar{L}\bar{F}V^+$ and $C_e \triangleq C + D\bar{L}\bar{F}V^+$.

**Lemma 1.** *The identity $y(t) = 0$ for all $t \geq 0$ implies that $w_1(t) = M_*x(t) = 0$ for all $t \geq 0$.*

A proof of Lemma 1 is given in Appendix B. Let $H_*$ be a full row rank matrix so that $\ker H_* = E\mathscr{V}^*$. ($H_*$ may be calculated by using the algorithm given in Appendix A).

**Theorem 1.** *$z(t)$ is FTR if, and only if,*

*(1) $E\mathscr{V}^* \cap B\ker D = E\mathscr{V}^* \cap B\bar{L}\ker D\bar{L}$, $\begin{pmatrix} BL^+ \\ DL^+ \end{pmatrix}$ is injective, and $\bar{\mathscr{V}}^* = \mathscr{V}^*$, or equivalently,*

*(2) $\operatorname{rank}\begin{pmatrix} H_*B \\ D \end{pmatrix} = \bar{m} + \operatorname{rank}\begin{pmatrix} H_*B\bar{L} \\ D\bar{L} \end{pmatrix}$ and $\operatorname{rank} V = \operatorname{rank}\bar{V}$.*

**Proof.** (*If*) If $y(t) = 0$, then, by Lemma 1, $w_1 = 0$. Thus, taking into account that $\operatorname{rank} V = \operatorname{rank}\bar{V}$ implies $\mathscr{V}^* = \bar{\mathscr{V}}^*$, and since $L\bar{L} = 0$, we obtain by (9) the identity

$$\begin{pmatrix} E\bar{V}\dot{w}_2 \\ 0 \end{pmatrix} = \begin{pmatrix} E\bar{V}Qw_2 \\ 0 \end{pmatrix} + \begin{pmatrix} B \\ D \end{pmatrix}\Psi^{-1}\begin{pmatrix} z \\ \bar{z} - \bar{F}w_2 \end{pmatrix}.$$

Furthermore, since $\ker H_* = E\mathscr{V}^* = E\bar{\mathscr{V}}^*$, then

$$0 = \begin{pmatrix} H_*B \\ D \end{pmatrix}\Psi^{-1}\begin{pmatrix} z \\ \bar{z} - \bar{F}w_2 \end{pmatrix}.$$

Therefrom, the first condition in the clause 2 of the theorem implies that $z(t) = 0$.