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A parameter identification method for continuous-time nonlinear systems and its realization on a Miura-origami structure

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ABSTRACT

Many mechanical systems are nonlinear and often high-dimensional. Constructing accurate models for continuous-time nonlinear systems calls for effectively identifying their parameters, whereas measurement noise and sensitivity to initial conditions make the identification challenging. This paper proposes a new parameter identification method for ordinary differential equations based on the idea of B-Spline Galerkin finite element. In this approach, the system's solution is globally constructed by a set of B-Splines. With Galerkin weak formulation, instead of taking analytical derivatives on basis functions, the differential terms are eliminated through integration by parts so that the measurement noise will not be amplified. Then least square algorithms can be adopted for solving the optimization problem to estimate the parameters. By solving two intractable testbed problems, the coupled Chua's circuits and the Tank reactor equations, we show that the new approach is effective and efficient in dealing with systems with high-dimensionality, complex nonlinearity, discontinuous input and output, and noisy data without specific pre-processing. In addition, this method is employed to identify the geometrical and mechanical parameters of a Miura-origami structure under base excitation, which possesses complex global nonlinearity, exhibits chaotic responses, and suffers from significant measurement noise. The proposed method gains success in dealing with this system; based on the identified parameters, the corresponding constituent force-displacement relation and the simulation results agree well with the experiments.

1. Introduction

Ordinary differential equations (ODEs) arise in many contexts of natural and social science for describing the temporal evolution of anything from a rocket launching to the spread of a disease, from electrical circuits to economic development [1]. Under some circumstances, the models are based on well-established physical principles, the parameters of ODEs can be determined from first principles or direct measurement. On the other hand, many ODEs are mathematical simplifications of actual systems or even data-driven models [2], their parameters cannot be determined through either of these approaches, which as a result, calls for parameter identification from experimentally measured data.

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The nonlinear least square (NLS) method is a straightforward approach for parameter estimation in ODEs [3,4]. Usually, closed-form solutions do not exist for a generic nonlinear ODE model. Hence, numerical integration scheme such as the Runge-Kutta algorithm is often used to obtain approximate solutions of the ODEs for a given set of parameters and initial conditions; then an iterative procedure is applied to find the optimal estimation of the unknown parameters that minimize the residual sum of squares of the differences between the experimental data and the numerical solutions [5]. Note that the system's initial conditions are always difficult to know accurately or are with noise. To solve this issue, one solution is to treat the initial conditions as an additional set of unknown parameters in the minimization scheme [6,7]. However, the NLS method calls for numerical integration during each iteration, which therefore induces problems including high computational costs and slow convergence, especially for those systems in high dimension and with complex nonlinearities.

An alternative to iterative numerical integration is to build a regression model using measured discrete-time data and their higher-order derivatives in a "direct approach" [8–10]. Here the derivatives can be approximated through various discrete-time algebraic operators, such as bilinear transform, forward/backward/central difference, or generalized finite difference operators [9–11]. Then the least squares method would be desirable to apply because of its good numerical properties and low computational burden, especially for fast or non-uniform sampling. Particularly, recent research has demonstrated the advantages of difference operators (i.e., the delta 'δ' operator) because the identified model based on discrete-time representation has structural similarity to the continuous-time ODE-model, and the identified parameters approach to their continuous-time counterparts as the sampling interval tends to zero [10,12,13]. These estimation methods have been applied in both linear and nonlinear continuous-time system identification [10,12–15]. Note that in order to derive high-order derivative, repeated numerical differences on data are unavoidable in these approaches, which may cause noise amplification and a biased least squares estimate. To overcome this deficiency, various denoising algorithms [16–18] and bias-removal methods [12,14] have been proposed. However, for those systems that are extremely sensitive to parameters, such as chaotic systems [10,19], the denoising and bias-removal approaches would not be effective. To identify and correct the errors and biases, the system's underlying dynamic behavior needs to be exploited, which on the other hand, is always cumbersome and case-dependent.

Another way to avoid iterative numerical integration is to represent the solution globally via a set of convenient basis functions. Then the numerical difference used in the abovementioned discretization-based methods can be replaced by analytical derivatives of the basis functions. The choice of basis is crucially important for taking derivatives, because a very accurate representation of the data may exhibit high-frequency small-amplitude oscillations that are catastrophic for derivative estimation. Generally, Fourier basis [20,21] is always adopted for periodic data, and B-spline basis [22,23] or wavelets basis [24,25] for open-ended data. In addition to the type of basis, deciding the number of bases is also a dilemma: the more basis functions, the better fit to the data, but with the risk of simultaneously fitting the undesired noise and amplifying the noise when taking derivatives; while with fewer basis functions, important smooth characteristics that we are trying to achieve may be missed [26]. Certain techniques have been proposed to tackle this dilemma, such as stepwise variable selection and variable-pruning methods [27] for adding or dropping basis functions, iteratively correcting and fitting the measurement [22], and roughness penalty for avoiding over-fitting [26]. For example, with the roughness penalty approach, although the number of basis functions is equal or greater than that of the knots, penalties will be applied to the roughness so that the fitted curve would emphasize more on the smooth characteristics of the data. However, a new problem arises that how much degree of roughness penalty should be applied; determining of which can be achieved through, e.g., the generalized cross-validation method [28], but is always computationally intensive.

Note that none of the parameter identification methods would be effective in all scenarios. In practice, some essential issues need to be taken into account when proposing a new method. First, the method has to be computationally efficient that each trial can be completed in a short time. Second, the method should be robust under noise, since a noisy measurement is always unavoidable. Third, the method is expected to be able to deal with complex nonlinearity and high dimensionality. In this paper, inspired by the B-Spline Galerkin finite element method [29,30], a new identification approach is developed. More specifically, in this method, although still relying on basis functions (B-Spline) to globally represent the solution, derivatives on basis functions are replaced by analytical integration-by-parts based on the Galerkin weak formulation. Hence, this method not only removes the need for time-consuming numerical integrations but also avoids numerical differences on discrete data or derivatives on basis functions that may induce undesired noise amplification. Note that similar ideas based on conventional Galerkin finite element method [31] have been proposed in [32], where piecewise-liner basis functions were adopted for constructing the solutions of linear systems (truss structures). In this research, via solving two numerical testbed problems and dealing with a practical Miura-origami (Miura-ori) dynamic problem, we show that the new method extends its applicability to high-dimensional systems with complex nonlinearity, and is both efficient and robust. Therefore, the method developed in this paper significantly advances the state of the art in terms of broad applicability, computationally efficiency, and robustness.

The rest of the paper is organized as follows. Section 2 introduces the system we are to identify and three problems that will be tackled by the new method, including two numerical testbed problems and a Miura-ori structure with strong nonlinearity. This is followed by detailed descriptions of the proposed method and the optimization procedures for linear and nonlinear systems in Section 3. The effectiveness of the method is verified in Section 4 on the two testbed problems. In Section 5, the method is applied to identify the geometric and physical parameters of a Miura-ori structure under dynamic excitations. Finally, summary and heuristic discussions are presented in Section 6.