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A new method for computation of eigenvector derivatives with distinct and repeated eigenvalues in structural dynamic analysis

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A B S T R A C T

Determining eigenvector derivatives is a challenging task due to the singularity of the coefficient matrices of the governing equations, especially for those structural dynamic systems with repeated eigenvalues. An effective strategy is proposed to construct a nonsingular coefficient matrix, which can be directly used to obtain the eigenvector derivatives with distinct and repeated eigenvalues. This approach also has an advantage that only requires eigenvalues and eigenvectors of interest, without solving the particular solutions of eigenvector derivatives. The Symmetric Quasi-Minimal Residual (SQMR) method is then adopted to solve the governing equations, only the existing factored (shifted) stiffness matrix from an iterative eigensolution such as the subspace iteration method or the Lanczos algorithm is utilized. The present method can deal with both cases of simple and repeated eigenvalues in a unified manner. Three numerical examples are given to illustrate the accuracy and validity of the proposed algorithm. Highly accurate approximations to the eigenvector derivatives are obtained within a few iteration steps, making a significant reduction of the computational effort. This method can be incorporated into a coupled eigensolver/derivative software module. In particular, it is applicable for finite element models with large sparse matrices.

1. Introduction

The computation of eigenvalue and eigenvector derivatives with respect to changes in structural design parameters is of wide practical importance in many fields, including structural dynamic optimization, system updating, damage detection, modification and identification [1–4]. However, determining eigenvector derivatives is still a challenging task due to the singularity of the coefficient matrices of the governing equations, especially for those structural dynamic systems with repeated eigenvalues.

To date, many methods have been developed for computing derivatives of eigenvalues and eigenvectors. The modal method was firstly derived by Fox and Kapoor [5] for the symmetric generalized eigenvalue problems. This technique

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requires the presence of knowledge for all eigenvectors, it is thus difficult for implementation and time-consuming for the analysis of large-scale structures. Based on the framework of this approach, Wang [6] improved the modal truncation method by using a residual static mode to approximate the contribution due to unavailable high-frequency modes. By means of the truncated modes to the eigenvector derivatives [7], the accuracy of computing the derivatives of eigenvalues and eigenvectors can be further improved. Afterwards, this approach was extended to investigate the symmetric and asymmetric systems with damping [8–10].

In an early effort, Nelson [11] proposed an efficient method to calculate the first-order derivatives of eigenvectors with distinct eigenvalues for the general real systems, by expressing the eigenvector derivatives as a particular solution and a homogeneous solution. In contrast to the modal method, the Nelson's method only requires considering both eigenvalues and eigenvectors. Friswell [12] extended the Nelson's method to find the second-order and higher-order derivatives of undamped systems. Besides, Friswell and Adhikari [13] further generalized the Nelson's method to calculate the eigenvector derivatives of symmetric and asymmetric systems. Guedria et al. [14] also applied the Nelson's method to compute the second-order derivatives of viscously damped systems.

Algebraic methods are another common approach that can be used to conduct the sensitivity analysis of mode shapes. Based on the rationale of this technique, a set of algebraic equations can be formulated using the derivatives of eigenvalue problems and the additional constraints can be determined from the derivatives of normalization. In the past, Lee and Jung [15] developed an algebraic method for the real symmetric eigenvalue problem with distinct eigenvalues. They [16] further extended the algebraic method to compute the eigenpair derivatives of symmetric systems with viscous damping. Besides, the algebraic approach was generalized for the eigensensitivities of asymmetric viscously damped systems [17–20]. Recently, Li et al. [21] extended this type of method to calculate the first-order and second-order eigenvector derivatives of undamped and damped nonlinear systems.

Furthermore, iterative methods are often used for the sensitivity analysis of eigensystems. Rudisill and Chu [22] presented an iteration method to find the first partial derivatives of eigenvalues and eigenvectors of self-adjoint systems. Andrew [23] offered a rigorous proof for the convergence of an iterative algorithm under the conditions mentioned in [22], and some refinements were also proposed in [24,25]. Based on the Subspace Iteration, Lanczos, Davidson and Arnoldi methods, the corresponding iterative algorithms for the analysis of eigensensitivities were established, respectively [26–30]. In addition, Alvin [31] introduced a preconditioned conjugate projected gradient (PCPG)-based technique for the analysis of eigenvalue problems. Xie [32] proposed a method that can be used to simultaneously compute the derivatives of several simple eigenvalues and the corresponding eigenvectors of unsymmetric damped systems. In the literature, some review studies [33,34] were also presented for the computation of derivatives of the general eigensystems with distinct eigenvalues.

It is worth noting that much research efforts [5–32] are only applicable to the case of distinct eigenvalue systems. However, there are many repeated or nearly equal eigenvalues in typical structural problems caused by two or more planes of the reflective or cyclic structural symmetry, e.g., wheelsets on trains. Besides, the repeated eigenvalues are far more likely to occur in optimized structures. The calculation of the derivatives of eigenvectors with repeated eigenvalues is more difficult, because the rank of the coefficient matrices of the governing equations is lower than those of the simple eigenvalues.

Generally, there are two key issues in the computation of eigenvector derivatives for simple and repeated eigenvalues by means of the improved Nelson's method. The first problem is how to find the particular solutions to the governing equations with a singular coefficient matrix, and the second one is how to determine the eigenvector derivatives for the given particular solutions. The previously proposed methods are mainly dependent on deleting rows and columns of the singular dynamic stiffness matrix [35–37] or using the bordered matrix method [38–40] to form a non-singular coefficient matrix, which requires re-ordering the matrix and destroying its sparsity. For example, Ojalvo [35] generated insufficient equations to determine the eigenvector derivatives form the particular solutions. Mills-Curran [36] and Dailey [37] considered the information from the second-order derivatives of eigenvalue problems to calculate the eigenvector derivatives of undamped systems with repeated eigenvalues. The case in which some of the first-order derivatives of repeated eigenvalues are not distinct was also considered [39,41,42]. Tang et al. [43,44] presented a method for the sensitivity analysis of repeated eigenvalues of general quadratic eigenvalue problems. Moreover, Xu and Wu [45] constructed a new normalization condition and developed an efficient method to compute the first-order derivatives of eigenvectors of symmetric viscously damped systems with distinct and repeated eigenvalues. More recently, Li et al. [46] proposed a new normalization for the left eigenvectors. Indeed, the left and right eigenvector derivatives can be computed in a parallel way for the asymmetric damped systems with distinct and repeated eigenvalues. They [47] also presented a combined normalization method to calculate the eigenvector derivatives of viscously damped systems with distinct and multiple eigenvalues.

On the other hand, Lee et al. [48] developed an algebraic method to consider the derivatives of eigensolutions of undamped eigensystems with multiple eigenvalues. Furthermore, Lee et al. [49] and Choi et al. [50] extended their algebraic method to the higher-order eigensensitivity of symmetric damped systems with repeated eigenvalues. Nevertheless, Wu et al. [40] pointed out that there exists a mistake in the derivation of the normalization of the systems with repeated eigenvalues [48].

Making use of a simultaneous iteration algorithm, Andrew and Tan [51,52] considered the eigenpair derivatives of repeated eigenvalues and its corresponding eigenvectors. Qian [53] proposed a numerical method to compute the first-order and higher-order derivatives of multiple eigenpairs of quadratic eigenvalue problems. Application of these methods to many models, however, is occasionally confronted to convergence problems. Recently, Wu et al. [54] presented a preconditioned conjugate gradient (PCG)-based iteration method to calculate the eigenvector derivatives of real symmetric eigen-