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Brief paper

On stability of multiobjective NMPC with objective prioritization☆

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ABSTRACT

This paper proposes a new multiobjective model predictive control (MO-MPC) of constrained nonlinear systems. According to objective prioritization, the MO-MPC problem is formulated as a lexicographic optimization problem. The optimal solutions are obtained by solving a hierarchy of single objective optimization problems. The conditions guaranteeing the recursive feasibility of the optimization problem and stability of the closed-loop system are derived, which depend only on the most important objective. Moreover, a suboptimal algorithm is presented to reduce the computational demand of MO-MPC. One characteristic of the proposed MO-MPC is that the given objective prioritization is automatically satisfied. The theoretical results are illustrated by a comparison study of an example.

1. Introduction

Multiobjective model predictive control (MO-MPC) has received attention recently, due to its ability to explicitly deal with system constraints and optimize a set of performance criteria systematically and simultaneously over a receding horizon (Maree & Imsland, 2014; Qin & Badgwell, 2003; Rawlings & Mayne, 2009). For most practical control problems, performance criteria often involve multiple conflicting control objectives, such as tracking, economical profit, environmental concerns, etc., which span different levels of relative importance. (See Flores-Tlacuahuac, Morales, & Rivera-Toledo, 2012, Zambrano & Camacho, 2002 and all the references therein.) Unlike the case of single objective MPC (SO-MPC) problems, in general, there is no unique (globally) optimal solution attainable to the MO-MPC problem (Chinchuluun & Pardalos, 2007 and Maree & Imsland, 2014). One feature of interest for the MO-MPC problem is to determine a Pareto optimal solution that satisfies the priorities of the multiple control objectives and guarantees the stability of the MO-MPC controller.

A practical approach for the MO-MPC is to form a scalar cost function being a weighted sum of individual cost functions with the weights that reflect the relative priorities of the multiple objectives. However, selecting a set of appropriate weights is a nontrivial task since reducing a weight on one objective and increasing the other does not necessarily lead to a proportional response in the face of constraints (see, e.g., Long & Gatzke, 2007, Tyler & Morari, 1999 and Vallerio, Van Impe, & Logist, 2014). Furthermore, for such systems as sewer network (Ocampo-Martinez, Ingimundarson, Puig, & Quevedo, 2008), certain objectives are only relevant under specific circumstances. Therefore, the selection of the weights associated with these objectives might not be appropriate when these objectives are irrelevant.

Lately, significant progress in MO-MPC has been reported. For instance, De Vito and Scattolini (2007) optimized linear MPC by minimizing the max of a finite number of objective functions. In Bemporad and Munoz de la Pena (2009) the MO-MPC was designed by minimizing a convex combination of different objective functions and stability of the closed-loop system was guaranteed for the convex combination that is close to the desired convex combination. For nonlinear systems, Magni, Scattolini, and Tanelli (2008) proposed a switched MO-MPC, where the stability was ensured by a state-dependent switch, i.e., the value of the activated cost function must be less than the one of the next activated cost function when the switch occurred. Müller and Allgöwer (2012) exploited the time-dependent switch of multiple cost functions to design MO-MPC of discrete-time nonlinear systems and made use of the average dwell-time method to achieve the stability of the proposed

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nonlinear MPC (NMPC). In Zavala and Flores-Tlacuahuac (2012), a utopia-tracking MO-NMPC was proposed to minimize the distance of a set of objective functions to its steady-state utopia point, where the stability was guaranteed by the terminal constraint and the assumption of strong duality. Benefits of this scheme are that the controller makes trade-off in the multiple objective functions automatically and the Pareto optimal set does not need to be computed on-line. Moreover, Maree and Imsland (2014) presented a dynamic utopia-tracking MO-NMPC scheme for economic optimization of cyclic processes, in which the recursive feasibility was derived by a cyclic terminal constraint; however, the stability of the resulting closed-loop system is still an open issue.

To handle priorities of multiple objectives effectively, the propositional logic and binary variables were used and therefore the original MO-MPC problem was transformed into a mixed integer nonlinear programming (MINP) (see, e.g., Bemporad & Morari, 1999, Long & Gatzke, 2005 and Vada, Slupphaug, Johansen, & Foss, 2001). In general, the MINP is harder to solve than nonlinear continuous optimization problems. By using the lexicographic optimization, Kerrigan and Maciejowski (2002) presented a general framework for design of MO-MPC with different prioritized objectives, where the MO-MPC problem was formulated by a sequence of single objective MPC problems according to the objective prioritization. Moreover, Ocampo-Martinez et al. (2008) designed a lexicographic MO-MPC for control of sewer network and Padhiyar and Bhartiya (2009) for profile control of distributed parameter systems. Zheng, Wu, Liu, and Ling (2010) proposed a new genetic algorithm to compute the lexicographic MO-NMPC actions. Some merits of the lexicographic MO-NMPC are that it can explicitly take into account the priorities of different objectives to be optimized, no arbitrary weights are used and the Pareto optimal set does not need to be computed at each time. To the best of our knowledge, however, no theoretical results of the feasibility of the lexicographic MO-NMPC problem, the stability and economic optimization have been reported in available literature.

Here we consider a class of MO-MPC problems of constrained nonlinear systems, where the objective functions of interest may be economic costs and conflicting, and are ordered according to their prioritization. The original MO-NMPC problem is then formulated as a lexicographic finite horizon optimal control problem (FHOCP), which is solved via a hierarchy of single objective FHOCPs. Two concepts of feasibility, i.e., hierarchical and horizontal feasibility, are introduced to achieve the recursive feasibility of the lexicographic FHOCP. The conditions for stability are obtained only using the most important objective function. The case of varying objective prioritization is discussed. In order to reduce the computational demand of solving the FHOCP online, a suboptimal prioritized MO-NMPC algorithm is presented. Then a well-known result that "*feasibility implies stability*" (Scokaert, Mayne, & Rawlings, 1999) for single objective MPC is regained. Two key features of the proposed NMPC are that the control actions explicitly rely on the objective prioritization and the stability is dependent only upon the most important objective function. The main contribution of this work is to present the feasibility and stability results of the MO-NMPC scheme subject to objective prioritization. Hence, it is a step forward in stability synthesis of MO-NMPC schemes that explicitly consider various priorities of multiple objectives.

2. Problem setup and preliminaries

Let $\mathbf{I}_{\geq 0}$ denote the set of non-negative integer numbers, $\mathbf{I}_{\geq a}$ be the set $\{i \in \mathbf{I}_{\geq 0} : i \geq a\}$ and $\mathbf{I}_{a:b}$ be the set $\{i \in \mathbf{I}_{\geq 0} : a \leq i \leq b\}$ for some $a \in \mathbf{I}_{\geq 0}$ and $b \in \mathbf{I}_{\geq 0}$. Label '$T$' in superscript denotes the transposition of a vector.

Consider the following discrete-time nonlinear system

$$x_{k+1} = f(x_k, u_k), \quad k \in \mathbf{I}_{\geq 0} \tag{1}$$

where $x_k \in R^n$ and $u_k \in R^m$ are the state and control vectors at sampling time k, respectively, and $f(\cdot, \cdot)$ is a locally Lipschitz function on its arguments with $f(0, 0) = 0$. The system is subject to constraints on the state and control

$$x_k \in X, \qquad u_k \in U, \qquad \forall k \in \mathbf{I}_{\geq 0} \tag{2}$$

where $X \subset R^n$ is a closed set and $U \subset R^m$ is a compact set, both of them containing the origin in their interior. Assume that the states are available for state feedback controllers.

Consider a finite sequence of future control at time k

$$\mathbf{u}_{k,N} = \{u_{0|k}, u_{1|k}, \ldots, u_{N-1|k}\} \tag{3}$$

where the prediction horizon $N \in \mathbf{I}_{\geq 1}$. For a given state x_k and sequence $\mathbf{u}_{k,N}$, the future state of the system at time $k+t$ predicted by using the model (1) at time k is denoted as $x_{t|k}$. Hence, $x_{t+1|k} = f(x_{t|k}, u_{t|k})$ with $x_{0|k} = x_k$. We consider l prioritized objectives of system (1), which are represented by objective cost functions

$$J_i(\mathbf{u}_{k,N}, x_k) = E_i(x_{N|k}) + \sum_{t=0}^{N-1} L_i(x_{t|k}, u_{t|k}), \quad i \in \mathbf{I}_{1:l} \tag{4}$$

where the stage costs $L_i : X \times U \to R$ and the terminal costs $E_i : X \to R$ are continuous on their arguments, $i \in \mathbf{I}_{1:l}$ and $l \in \mathbf{I}_{\geq 2}$. In this paper, the objective functions (4) are assumed to be conflicting and there is no solution optimizing all objectives at the same time.[2] Therefore, additional mechanisms must be used to balance these objectives. Here we exploit the objective prioritization to compute the optimal control sequence.

Without loss of generality, we assume that the objective functions are in the order of importance so that J_1 is the most important and J_l the least important to decision makers. According to this objective prioritization, we define a prioritized multiobjective FHOCP

$$\min_{\mathbf{u}_{k,N}} J(\mathbf{u}_{k,N}, x_k) \tag{5a}$$

$$\begin{aligned} \text{s.t.} \quad & x_{t+1|k} = f(x_{t|k}, u_{t|k}), \quad x_{0|k} = x_k \\ & x_{t+1|k} \in X, \qquad u_{t|k} \in U, \quad t \in \mathbf{I}_{0:N-1} \end{aligned} \tag{5b}$$

where the current state $x_k \in X$, decision vector $\mathbf{u}_{k,N}$ is given by (3) and $J(\mathbf{u}, x)$ is the objective function vector

$$J(\mathbf{u}, x) = [J_1(\mathbf{u}, x), J_2(\mathbf{u}, x), \ldots, J_l(\mathbf{u}, x)]^T \tag{6}$$

which maps the constrained control sequence $\mathbf{u}$ and current state x to a set of values of l objective functions (4). Here the optimization of the vector is defined in the sense of the dominance notion (Marler & Arora, 2004), i.e., an objective function vector $J(\mathbf{u}^*, x)$ is non-dominated if and only if there does not exist another vector $J(\mathbf{u}, x)$ such that $J(\mathbf{u}, x) \leq J(\mathbf{u}^*, x)$ with at least one $J_i(\mathbf{u}, x) < J_i(\mathbf{u}^*, x_k)$.

In SO-NMPC, the optimal control sequence is computed by minimizing a single objective function at each time. In contrast to SO-NMPC, the MO-NMPC must minimize l different (conflicting) objective functions at each time. Therefore, there is typically no single optimal solution but rather a set of possible non-dominant solutions of equivalent quality (Abraham & Jain, 2005). The Pareto optimality is an effective measure of the equivalent quality in multiobjective optimization problems (Chinchuluun & Pardalos, 2007; Ehrgott, 2005). Let $\mathbf{u}^*_{k,N}$ be one of the Pareto optimal

[2] In the general case when some cost functions are consistent, we combine them into a single one.