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Brief paper

Leader-following rendezvous with connectivity preservation and disturbance rejection via internal model approach☆

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ABSTRACT

Most of the recent results on the leader-following rendezvous problem focus only on the deterministic multi-agent systems without external disturbances and plant uncertainties. In this paper, we will present a novel distributed internal model approach to further study the leader-following rendezvous problem for double-integrator multi-agent systems subject to both external disturbances and plant uncertainties. We provide both the distributed full state feedback control and the distributed partial state feedback control without velocity measurement. In both control schemes, we will give the suitable distributed internal model to convert the rendezvous problem into the stabilization problem with connectivity preserving for their augmented systems. We stabilize the corresponding augmented systems by the high gain feedback control based on a new potential function. Comparing with some recent results, our design can handle a large class of reference signals, external disturbances, and plant uncertainties.

1. Introduction

Over the past decade, the cooperative control of multi-agent systems has attracted extensive attention of the control community. Among all the cooperative control problems, the rendezvous problem is one of the most important topics. In contrast to the consensus problem, in which the connectivity of the network topologies at any time instant are usually defined in advance, see Hu and Hong (2007), Jadbabaie, Lin, and Morse (2003), Olfati-Saber and Murray (2004) and Ren and Beard (2008), the rendezvous problem requires the controller able to maintain the connectivity of the network topologies. Such a network is usually described by a time-varying graph, in which the given edge between any two agents is acted if and only if the distance between these two agents is less than some positive real number that is called the sensing range. It is now well known that the connectivity can be preserved via some suitable potential function based approach (Ji & Egerstedt, 2007). For different objectives, the rendezvous problem can be divided into two classes: leaderless and leader-following. While the leaderless rendezvous problem aims to make the positions of all agents approach a same but usually unknown location, the leader-following rendezvous problem further requires the positions of all agents asymptotically track a special trajectory which is called the leader system. So far, the leaderless rendezvous problem has been widely studied for single-integrator multi-agent systems, see Dimarogonas, Loizou, Kyriakopoulos, and Zavlanos (2006), Ji and Egerstedt (2007), Yang et al. (2010), and Zavlanos and Pappas (2007) and double-integrator multi-agent systems, see Su, Wang, and Chen (2010). The leader-following rendezvous problem has also received a lot of attentions for single-integrator multi-agent systems, see Gustavi, Dimarogonas, Egerstedt, and Hu (2010) and Ji, Ferrari-Trecate, and Buffa (2008) and double-integrator multi-agent systems, see Dong and Huang (2013) and Su et al. (2010).

Most of these recent results focus only on the deterministic multi-agent systems without external disturbances and plant uncertainties. Particularly, paper (Su et al., 2010) first studied the leader-following rendezvous problem for double-integrator multi-agent systems with the leader system being also the double-integrator. More recently, paper (Dong & Huang, 2013) further considered the problem for the same class of double-integrator multi-agent systems as that in Su et al. (2010) but with a more general class of the leader system and subject to a class of external disturbances. Both the reference tracking signal and the external disturbance are assumed to be generated by an autonomous linear system that is called the exosystem or the

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leader system. As a result, this class of disturbances can be arbitrary without assuming to be bounded. Paper (Dong & Huang, 2013) provided a feedforward design method with a distributed observer approach to reject the disturbances. The design method, however, explicitly relies on the solution of the regulator equations (Francis & Wonham, 1976). Thus, it is not applicable to the system with plant uncertainties.

In this paper, we will present a novel distributed internal model approach to study the leader-following rendezvous problem for double-integrator multi-agent systems subject to both external disturbances and plant uncertainties. We will consider both the distributed full state feedback control and the distributed partial state feedback control without velocity measurement. In both control schemes, we will give the suitable distributed internal model to form the corresponding augmented system. For the full state feedback case, the augmented system is composed of the multi-agent system and the internal model, while for the partial state feedback control case, the augmented system is composed of the multi-agent system, the internal model and an input driven filter. By the output regulation theory (Huang, 2004), the rendezvous problem is converted to the stabilization problem with connectivity preserving for these augmented systems. We then successfully stabilize the corresponding augmented systems by the high gain feedback control based on a new potential function. The major novelty, comparing with the recent results (Dong & Huang, 2013; Su et al., 2010), lies in that our design can handle a large class of reference signals, external disturbances, and plant uncertainties, simultaneously. A detailed comparison will be shown in Remark 1.

It is worth mentioning that the internal model based control for multi-agent systems with fixed and switching network topologies are recently presented in Hong, Wang, and Jiang (2013), Su, Hong, and Huang (2013), Wang and Han (2011), Wieland, Sepulchre, and Allgöwer (2011) and Yu and Wang (2013). Such a problem is called the cooperative output regulation problem, and can be viewed as a generalization of the leader-following consensus problem. However, the controllers in Hong et al. (2013), Su et al. (2013), Wang and Han (2011), Wieland et al. (2011) and Yu and Wang (2013) are all linear, and hence are not able to maintain the connectivity. In contrast, we have to resort the potential function based nonlinear control law in this paper.

The rest of this paper is organized as follows. In Section 2, we give our problem formulation as well as some preliminary results. In Sections 3 and 4, we study the solvability of our problem via the distributed full state feedback control and the distributed partial state feedback control without velocity measurement, respectively. We provide an example in Section 5. Finally, in Section 6, we finish the paper with conclusions.

Notation: Given the column vectors a_i, $i = 1, \ldots, s$, we denote $\mathrm{col}(a_1, \ldots, a_s) = [a_1^T, \ldots, a_s^T]^T$. Given two matrices A and B, the symbol $A \otimes B$ represents the Kronecker product of A and B. Given a finite set S, the symbol $|S|$ denotes the cardinality of S. Given a symmetric matrix A, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the minimum and maximum eigenvalues of A, respectively. Given two symmetric matrices A and B, the symbol $A \geq B$ means the matrix $A - B$ is positive semi-definite.

2. Problem formulation and preliminaries

Consider the class of double-integrator multi-agent systems

$$\begin{aligned} \dot{q}_i &= p_i + d_{1i}, \\ \dot{p}_i &= u_i + d_{2i}, \quad i = 1, \ldots, N, \end{aligned} \tag{1}$$

where $q_i \in \mathbb{R}^n$ denotes the position of the ith agent, $p_i \in \mathbb{R}^n$ denotes the velocity of the ith agent, and $d_{1i} \in \mathbb{R}^n$ and $d_{2i} \in \mathbb{R}^n$ represent the external disturbances in the plant. Let $q_0(t) \in \mathbb{R}^n$ be the reference trajectory. Both the trajectory signal of the leader $q_0(t)$ and the disturbance signals $d_{1i}(t)$ and $d_{2i}(t)$ are assumed to be generated by the linear exosystem

$$\dot{v} = Sv, \tag{2}$$

with $q_0 = F(w)v$, $d_{1i} = E_{1i}(w)v$, $d_{2i} = E_{2i}(w)v$, where $v \in \mathbb{R}^{n_v}$, $S \in \mathbb{R}^{n_v \times n_v}$, $E_{1i}(w), E_{2i}(w), F(w) \in \mathbb{R}^{n \times n_v}$, and $w \in \mathbb{R}^{n_w}$ is an uncertain parameter vector. The exosystem (2) can generate a large class of practical reference signals such as step functions, ramp functions, polynomial functions, exponential functions, and sinusoid functions, as well as their products and combinations. It is common studied in the output regulation theory, see Huang (2004).

The plant (1) and the exosystem (2) together are viewed as a multi-agent system of $N + 1$ agents with the exosystem as the leader and all the subsystems of (1) as the followers. Motivated by Ji and Egerstedt (2007), with respect to (1) and (2), we can define a time-varying graph $\bar{\mathcal{G}}(t) = \{\bar{\mathcal{V}}, \bar{\mathcal{E}}(t)\}$, where $\bar{\mathcal{V}} = \{0, 1, \ldots, N\}$ with the node 0 associated with the exosystem and the other N nodes associated with the N followers, respectively, and $\bar{\mathcal{E}}(t) \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$ by the following rules: given any $r > 0$, for any $t \geq 0$, $\bar{\mathcal{E}}(t) = \{(i, j) : i, j \in \bar{\mathcal{V}}\}$ is defined such that

1. for any $i \neq j$, $i = 0, 1, \ldots, N$, $j = 1, \ldots, N$, $(i, j) \in \bar{\mathcal{E}}(t)$ if and only if $\|q_i(t) - q_j(t)\| < r$;
2. $(i, 0) \notin \bar{\mathcal{E}}(t)$ and $(i, i) \notin \bar{\mathcal{E}}(t)$ for any $t \geq 0$ and any $i = 0, 1, \ldots, N$.

We define the neighbor set of the ith agent at time t as $\bar{\mathcal{N}}_i(t) = \{j : (j, i) \in \bar{\mathcal{E}}(t)\}$. Then we will consider a distributed dynamic full state feedback control law of the form

$$\begin{aligned} u_i &= k_{1i}(\zeta_i, p_i, q_i - q_j, j \in \bar{\mathcal{N}}_i(t) \cap \bar{\mathcal{N}}_i(0)), \\ \dot{\zeta}_i &= h_{1i}(\zeta_i, p_i, q_i - q_j, j \in \bar{\mathcal{N}}_i(t) \cap \bar{\mathcal{N}}_i(0)), \quad i = 1, \ldots, N, \end{aligned} \tag{3}$$

where k_{1i} and h_{1i} are some nonlinear functions vanishing at the origin, and $\zeta_i \in \mathbb{R}^{n_{\zeta_i}}$ with n_{ζ_i} to be defined later, and a distributed dynamic partial state feedback control law of the form

$$\begin{aligned} u_i &= k_{2i}(\zeta_i, q_i - q_j, j \in \bar{\mathcal{N}}_i(t) \cap \bar{\mathcal{N}}_i(0)), \\ \dot{\zeta}_i &= h_{2i}(\zeta_i, q_i - q_j, j \in \bar{\mathcal{N}}_i(t) \cap \bar{\mathcal{N}}_i(0)), \quad i = 1, \ldots, N, \end{aligned} \tag{4}$$

where k_{2i} and h_{2i} are some nonlinear functions vanishing at the origin, and $\zeta_i \in \mathbb{R}^{n_{\zeta_i}}$ with n_{ζ_i} to be defined later. Both controllers (3) and (4) are distributed because the control input of the ith agent can only access the information of itself and its neighbors. In particular, in this paper, we assume that only those agents whose neighbors contain the leader can access the trajectory of the leader, while the velocity of the leader is unknown for any follower. Now we define the leader-following rendezvous and disturbance rejection problem for system (1) as follows.

Problem 1. Given the multi-agent system composed of (1) and (2), and any $r > 0$, find a distributed control law of the form (3) or (4) such that, for all $w \in \mathbb{R}^{n_w}$ and all initial states that make the initial graph $\bar{\mathcal{G}}(0)$ connected in the sense that every node i, $i = 1, \ldots, N$ is reachable from the node 0, the closed-loop system has the following two properties: (i) $\bar{\mathcal{G}}(t)$ is connected for all $t \geq 0$; (ii) $\lim_{t\to\infty}(q_i(t) - q_0(t)) = 0$, $i = 1, \ldots, N$.

Remark 1. It is worth relating our problem formulation to those of two recent papers (Dong & Huang, 2013; Su et al., 2010), which also consider the leader-following rendezvous problem of double integrator multi-agent systems. Here our problem is much more challenging than those in Dong and Huang (2013) and Su et al. (2010) at least from the following aspects.