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An approach to optimal semi-active control of vibration energy harvesting based on MEMS

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ABSTRACT

In this paper the energy harvesting problem involving typical MEMS technology is reduced to an optimal control problem, where the objective function is the absorption of the maximum amount of energy in a given time interval from a vibrating environment. The interest here is to identify a physical upper bound for this energy storage. The mathematical tool is a new optimal control called Krotov's method, that has not yet been applied to engineering problems, except in quantum dynamics. This approach leads to identify new maximum bounds to the energy harvesting performance. Novel MEMS-based device control configurations for vibration energy harvesting are proposed with particular emphasis to piezoelectric, electromagnetic and capacitive circuits.

1. Introduction

Among all the possible sources for energy harvesting, mechanical vibrations are widely present and represent a subject that experimented a constant growth in the last years [1–4]. The present paper is devoted to the analysis of the control models of MEMS that absorb energy from vibrating sources, but the general form of this analysis can be used for a wider range of energy extraction problems.

One of the first approaches to the problem of optimization of energy harvesting for MEMS is presented in [5]. This work is based on a parameter optimization approach, which modifies, at design stage, the parameters of the device in order to maximize the harvested energy. This approach is followed by many others [6–10], and different devices and parameters have been considered. We call this technique the *classical optimization method*, and it lies outside of the field of control. One recognizes in [11–15] seminal contributions to the problem of energy harvesting control. In these works, one of the most important results of Optimal Control Theory, the LQG [16], is applied to construct optimal feedback control laws for optimizing the harvested power. These approaches have the merit of considering the stochastic nature of the vibrations pumping energy into the MEMS, and provide simple linear control laws. However, these results are obtained introducing an active control which admits possible external power injections. On the other hand, the construction of an LQG control requires the presence of strictly convex terms in the energy functional [16] which is not general for all cases.

In this paper the proposed strategy for controlling vibration energy harvesting by MEMS is indeed performed through semi-active passive devices. Additionally in the present paper we do not make any use of the *clipping* technique, which constraints *a posteriori* the control variable to satisfy the saturation bounds which characterize semi-active controls [17].

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The point of view illustrated in the present paper contains several original contributions: (i) a general model for an optimal vibration energy harvesting is presented, (ii) this problem is proposed in its most natural form, which is that of nonlinear semi-active passive control of MEMS, that has not been yet proposed in the literature, (iii) the problem is here attacked with an optimization method not yet considered in the field of engineering, the Krotov's control.

This approach has its appeal because it is able to produce absolute maxima for the energy extractor performances only regulating the circuit dissipators (resistors), that is the control of the energy harvester produces the maximum possible power from the energy source. The Krotov's method is probably one of the most promising control strategies that has been applied up today only to special problems of quantum mechanics, but that indeed has very interesting perspectives also in engineering applications.

2. MEMS energy harvesting from a vibratory source: physics and control

In this section the primary energy source is represented by a vibrating surface to which is attached a device that undergoes the prescribed oscillations of the surface. This example, at least conceptually, includes different physical mechanisms of energy harvesting. Basically the device consists of a sprung mass attached by an elastic element to the vibrating surface, and the mechanical/electrical conversion is based on three alternative physical principles: electromagnetic, piezoelectric and capacitive. These schemes form the basis of actual MEMS technology for energy harvesting.

In the following three sections, we consider in detail the physical governing equations of these electro-mechanical circuits and the associated optimal control harvesting problem.

2.1. Electromagnetic energy harvesting

This kind of devices use a permanent magnet to subtract energy from a vibrating surface. The magnet with mass m is attached to the surface with an elastic element and its motion produces a change of the magnetic flux through a coil in a circuit as it is depicted in Fig. 1. The physical basis for this device to extract energy from the environment is twofold. First, the sprung mass, under the action of a moving basement, undergoes the magnetic force induced by the coil (Lorentz force) $F_B \sim -Bi$, where i is the current through the resistor R. It represents an energy storage in some auxiliary circuit not detailed here, that is percept as an energy dissipation for the main circuit represented in Fig. 1. The magnetic field B is supposed to be constant in time. The second system, the circuit which produces the effective energy dissipation, is dominated by the dynamics of the current induced in the coil which f.e.m. is

$$\text{f.e.m.} = -\frac{\partial \Phi}{\partial t} \tag{1}$$

where Φ is the magnetic flux through the coil, and is composed by the self inducted flux and the flux due to the magnetic mass

$$\Phi = Li - \Phi_{\text{ext}}$$

and

$$\text{f.e.m.} = -L\frac{di}{dt} + \frac{d\Phi_{\text{ext}}}{dz}\frac{dz}{dt}$$

where L is the system's inductance. In this model we suppose that the relative displacement of the mass z with respect to the basement is such that the term $d\Phi_{\text{ext}}/dz$ can be considered as a constant positive value k_t. Now, we can write the Kirchhoff law of the circuit as

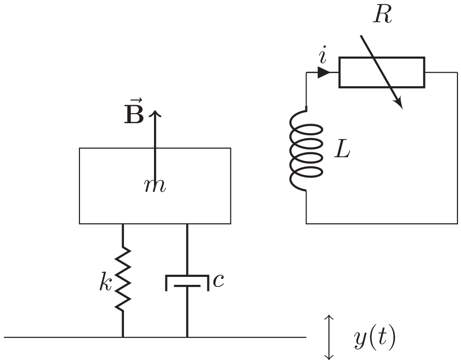

Fig. 1. Simplified sketch of an electromagnetic energy harvester.